ABSTRACT
A physical modeling synthesizer is developed and used to parametrically represent digital audio recordings of clarinet soloists. Empirical data, in the form of acoustic impedance measurements, are incorporated in the model definition. Reed dynamics are also included. Algorithms which calculate control parameters from source recordings are presented and examples of both the impedance measurements and the synthesis output are provided. The synthesis results give further support to the idea that music can, in certain circumstances, be adequately represented by a few low bandwidth control parameters. Additionally, the governing equations of clarinet physical modeling are briefly reviewed.

Index Terms—music, modeling, signal synthesis

1. INTRODUCTION
Physical modeling is a method for synthesizing lifelike musical sounds. As opposed to additive synthesis, subtractive synthesis, or granular synthesis, physical modeling relies on a numerical simulation of the physics underlying instrument-player systems [1]. As computing power has increased, this approach has become increasingly viable. The main advantage of the physical modeling paradigm is that it naturally incorporates many aspects of music which can be difficult to realize with conventional synthesizers. For instance, given the same set of control parameters and a sufficiently detailed model, oscillations build up and decay in the model just as they would in a real musical instrument.

A further advantage of physical modeling is that musical sounds are parametrically represented in terms of what a player does. These control parameters, such as blowing into a mouthpiece or pressing keys require significantly less bandwidth for storage than CD quality digital audio. Some promise exists, therefore, to create an extremely compact form for musical audio/data that simultaneously allows for high-fidelity reproduction and is based upon musically relevant gestures.

This paper outlines a preliminary attempt to construct such a system in the case of monophonic clarinet music. There are two aspects of this work: the implementation of a physical model, and the development of algorithms which can extract appropriate control parameters from source audio files. The clarinet is one of the most studied instruments from the point of view of physical modeling, which provides ample context for the present study.

2. PHYSICAL MODEL
The basic equations governing the pressure oscillations of a clarinet were presented by Schumacher [2]. Let $p_b(t)$ and $u(t)$ equal, respectively, the acoustic pressure and acoustic volume velocity just inside the mouthpiece of the clarinet. Call the pressure inside the mouth of the player (the blowing pressure) $p_m(t)$.

\[ p_b = \int_0^\infty h(t-\tau)u(\tau)d\tau \]  
\[ u = g\left(p_m - \int_0^\infty h(t-\tau)u(\tau)d\tau\right) \]  

where
\[ g(\Delta p) = U_M \frac{3\sqrt{3}}{2} \left(1 - \frac{\Delta p}{p_{ext}}\right) \left(\frac{\Delta p}{p_{ext}}\right)^{1/2} \]  
\[ \Delta p = p_m - p_b \]  

These are nonlinear integral equations involving the acoustic variables. Equation (3) can be derived from the Bernoulli equation [3] [4] while the constant parameters $U_M$ and $p_{ext}$ are, respectively the maximum volume flow and the extinction threshold (where the flow is cutoff due to the reed pressing against the mouthpiece lay). The behavior of the pressure flow characteristic is shown in Fig. 1.

McIntyre, Schumacher, and Woodhouse [5] demonstrated how the above equations fit into a more general picture of sound generation in a variety of instruments. The MSW model consists of a nonlinear system and a linear system in cascade with a feedback tap and an external input representing the energetic contribution by the player. Here, the nonlinearity corresponds to the reed pressure-flow characteristic $g$ while the
linear system corresponds to the impulse response of the clarinet bore \( h(t) \), equal to the Fourier transform of the acoustic impedance \( Z(j\omega) \) of the bore. It is noted that the bore impulse response will change depending on the fingering that the player applies. This is the mechanism that allows different notes to sound. The filter in (1) must therefore be understood as a time-varying filter. In total, there are 46 playable notes on a standard clarinet.

We distinguish our physical model as empirical because it employs acoustic measurements taken on an actual clarinet. In a later section, the measurement of the acoustic impedance \( Z(j\omega) \) of the instrument air column is discussed. The collection of 46 impedance curves and impulse responses provides a complete description of the clarinet bore for physical modeling.

The equations above can be written discretely as follows. Let \( L \) denote the length in samples of the bore impulse response. For the simulations presented, \( L = 7000 \) with a sampling rate \( f_s = 44100 \) Hz. The coefficients of the filters for different notes are stored in column vectors \( h_i \) with the subscript \( i \in (0, 46) \) denoting the different bore responses (\( i = 0 \) is a zero valued filter which can be used during silent passages or rests). A state vector, containing the past values of \( u \), is created.

\[
U_n = \begin{bmatrix} u[n] \\ u[n-1] \\ \vdots \\ u[n-L+1] \end{bmatrix}
\] (5)

The bore pressure, at any sample, is then equal to an inner product.

\[
p_b[n] = h_i^T U_{n-1}
\] (6)

\[
u[n] = g(p_m - h_i^T U_{n-1})
\] (7)

### 2.1. Reed Dynamics

The pressure-flow characteristic \( g \) does not address the dynamical aspects of the reed. Simulation of the clarinet using only (2) carries the implicit assumption that the reed-valve acts in a completely memoryless fashion. In actuality, the reed is a cantilevered beam system which can potentially oscillate in a variety of vibratory modes and which has associated with it material properties such as mass and stiffness. Typically, the tip of the reed is assumed to obey the equations of a damped harmonic oscillator. In our simulations, the reed dynamics are included according to the prescription given by [6]. The variable \( x \) is intended to be the displacement of the reed tip from its resting position.

\[
\frac{1}{\omega_r^2} \frac{d^2 x(t)}{dt^2} + \frac{q_r}{\omega_r} \frac{dx(t)}{dt} + x(t) = p_b(t)
\] (8)

In [6] the second order system of the reed is approximated by an IIR filter (9). The filter coefficients depend upon the desired resonance frequency and quality factor of the reed. The pressure flow characteristic can be written as a function of the reed displacement (10).

\[
x[n] = b_1 p_b[n-1] + a_1 u[n-1] + a_2 u[n-2]
\] (9)

\[
u = \Theta(1 - p_m + x) \times (1 - p_m + x) \times \text{sgn}(\Delta p) \sqrt{|\Delta p|}
\] (10)

### 2.2. Impedance Measurements

The clarinet physical model described herein is empirical in that it incorporates data derived from experiment. Specifically, the experimental data consists of 46 impulse responses which were calculated from impedance measurements taken on a Selmer clarinet for 46 standard fingerings applied.

Impedance measurements were conducted with the piezoelectric disk method [7]. There are, in the literature, a large number of techniques for measuring acoustic impedance. We found the piezo-disk method to be the most satisfactory. Custom impedance heads were constructed from Mouser piezoelectric benders fastened to short sections of 0.5 inch diameter PVC pipe. Pressure recordings were taken with Isomax B6 Lavaliere microphones. The generation of test signals and analysis was done with a Stanford Research Systems SR780 Frequency Analyzer. Calibration routines and Fourier inversion were accomplished with a collection of Matlab functions.

A number of details are involved in the process of going from impedance measurements to discrete filter impulse
responses which can be used in a physical model. Specifics to our approach may be found in [8]. The original paper by Benade and Ibisi [7] addresses calibration issues. Additional information may be found in [9]. We do not delve too deeply into this matter here because the length would be prohibitive and it is not immediately relevant to the implementation of the physical model. The salient point is that the acoustic measurements provided us with a complete description of the clarinet instrument air column.

3. CONTROL PARAMETERS

Given a digital audio recording of a solo clarinet, our task is to extract from it a set of control parameters that allow the physical model to synthesize identical musical content. The control parameters may be conceptually divided into two sets. One set concerns the notes that were played and at what times they occurred (equivalently, which fingering the player applied at any time). This information is captured in a matrix of note onsets and pitches. Note segmentation can be performed with cepstral or time-frequency analysis and we refer to [10]. The other set consists of the player blowing pressure and parameters relating to the embouchure. These are generally more difficult to infer from a recording.

Due to the inherent nonlinearities of the clarinet model, our potential lack of knowledge about the acoustic environment in which a recording was made, and the noninvertibility of the bore impulse response $b(t)$ parameter estimation becomes a difficult problem. However, it is clear that to recreate the musical dynamics of a given recording the envelope of our synthesized output should closely match the envelope of the original. Moreover, the envelope of the output of the physical model is largely determined by the blowing pressure. The blowing pressure control parameter, therefore, has been set equal to a scaled version of the envelope of the recording we are attempting to recreate.

It is an easy matter to devise an algorithm for computing the envelope of any signal. Positive results were obtained by passing the absolute value of solo clarinet recordings through a low pass FIR filter. The envelope was calculated as in (11). The filter coefficients of the lowpass filter $h_{lp}$ were determined with the Matlab function `fir1`. An example of the output of this algorithm is shown in Fig. 3.

$$\text{env } p = h_{lp} * |p|$$  

One caveat is worth noting. Due to reverberant effects, the sound field in a room will decay quickly but not instantaneously after a clarinetist stops playing a note. Even under anechoic conditions the instrument itself will have a finite ring-down controlled by the size of the support of the bore impulse response. A microphone will capture this and the reverbation will ultimately reflect in the control parameters if the blowing pressure is found by the envelope method above, possibly eliding the player’s articulation. In ongoing work we remove this effect.

4. SYNTHESIS RESULTS

Synthesis examples from our empirical physical model are presented in this section. Control parameters were computed as outlined in the preceding section. The source recordings were of a professional clarinetist playing in anechoic conditions. A plot of the time domain waveforms of the original source recording and the synthesized output of the physical model are given in Fig. 4. Five notes are represented in these waveforms. Time-frequency distributions of the same five notes are given in Fig. 5.
5. CONCLUSION

A system for representing solo clarinet music through physical modeling has been demonstrated. In particular, we have developed software in two complementary areas: the physical modeling numerical simulation itself, and algorithms which take solo clarinet audio recordings and furnish correct control parameters for the physical model. Furthermore, the model is empirical in that precise acoustic measurements are incorporated into its definition. The physical modeling based representation is highly compact. The impulse response data, the note onsets and pitches and the blowing pressure parameter (sampled at a rate of \( \approx 50 \text{ Hz} \)) are all that are needed for reliable synthesis.

A number of topics relating to the present work may be pursued in the future. The empirical method we have followed in constructing the physical model can be extended to almost any other wind instruments. The data accompanying such a project would likely be an invaluable aid in understanding differences and similarities amongst instrument families. Alternatively, the clarinet model may be improved upon by including more details, such as the effect of the player’s vocal tract and instrument radiation transfer functions.

6. REFERENCES


