

IN-SITU CALIBRATION OF ACCELEROMETERS IN BODY-WORN SENSORS USING QUIESCENT GRAVITY

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ABSTRACT

As the cost, size and power required by sensor devices decrease, an increasing range of applications are possible. We focus on the application of tracking accelerometer data from body worn sensors over long durations in health monitoring applications. Body worn sensors must be compact for the convenience of the patient and low power to support extended operation, but these benefits can come at the cost of accuracy. To mitigate this loss of accuracy we first examine the ability of calibration to improve accuracy. Next we propose an in-situ method for calibrating the accelerometers using the quiescent acceleration due to gravity as a calibration signal. This in-situ calibration uses only the stored data from the duration the sensor is worn by the patient and does not require any extra procedures or measurements from the physical sensor devices. Compared to a manual three-axis calibration technique, the proposed calibration can be applied to data without the need for specific calibration procedures or even access to the original sensor and provides comparable or better accuracy.

Index Terms— calibration, accelerometer, three axis sensor, ellipsoid

1. INTRODUCTION

Three axis accelerometers are microelectromechanical systems (MEMS) that can provide a low cost solution to measure both the magnitude and orientation of a device's acceleration. A key point is that on the Earth's surface gravity produces a constant acceleration of 9.81m/s^2 in the downward direction which can be used to estimate a device's vertical orientation. Accelerometers have become ubiquitous in hand-held electronic devices where they can be used to reorient the screen according to how the device is held, and for gesture recognition to augment the user-interface abilities of the device [1]. Additionally, compact accelerometers have been used in health monitoring applications [2, 3, 4] to collect data

over long durations that would not be practical in a clinical setting. In both applications an accurate estimate of the magnitude and direction of the acceleration is key. However, in low cost devices calibration errors can cause significant error, while manual calibration procedures would add significant cost to the device and may not even be possible for some in-situ applications where a user or patient cannot be expected to carryout a specific procedure with the needed accuracy.

Many different three axis calibration techniques have been proposed for the needs of different applications. Many times it is sufficient to assume that the three sensors for each axis are orthogonal and calibrate only the sensors' gains and offsets [5, 6]. Most prior calibration techniques require a specific procedure in which the sensor is put in multiple known orientations, and a model for the calibration errors allows using the observed data from the known orientations to estimate the calibration correction factors [7, 8, 9]. The key distinction of the proposed calibration technique is that it automatically detects durations during normal operation when the acceleration due to gravity is not obscured by other movements, and uses these durations to automatically calibrate the three axis sensors without the need for any specific procedures or known orientations. Additionally, as opposed to [2] the proposed technique is not iterative. Ellipsoid based fitting has been proposed for three-axis magnetic sensors [10, 11] and for accelerometers when offline procedures are used to collect data when the device is at rest [12]. However, the proposed calibration is calculated directly from the arbitrary observed data itself after it is downloaded from the sensor. Calibration takes into account the entire duration of data and calculates a single set of calibration parameters without needing any access to the physical device.

The organization of this paper is as follows: Section 2 formalizes the problem of three axis calibration; Section 3 gives the proposed calibration technique; Section 4 gives the results of the proposed technique as compared to a typical calibration procedure using known orientations; and concluding remarks are given in Section 5.

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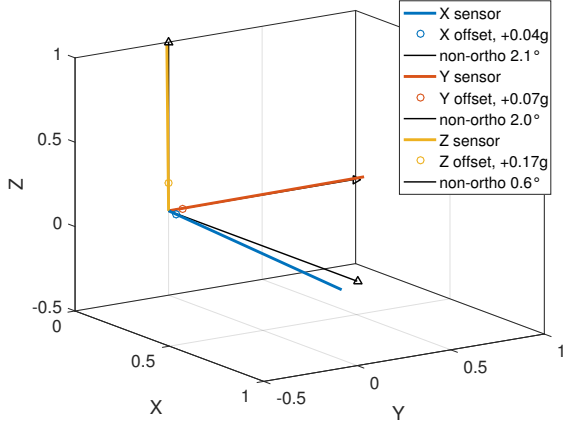


Fig. 1. Calibration gives the orientation misalignment of the X , Y and Z axis sensors as well as the gain and offset for each sensor.

2. CALIBRATION METRICS

The calibration error of a three axis accelerometer is modeled by three components for each of the three sensors corresponding to the X , Y and Z axes: first, each sensor has a DC offset that is always present. Even in the absence of any net acceleration the offset still causes the sensor to register a measurement. Next, each sensor is modeled as linear with a gain that can vary from unity. Gain is important because a relative difference in the gain between sensors for the different axes can cause the measured magnitude of a constant acceleration to vary depending on the device's orientation. Offset and gain are represented by the values b_x , b_y and b_z , and the factors g_x , g_y and g_z for the sensors corresponding to the X , Y and Z axes respectively. This means that if the projection of the true acceleration vector onto to orientation of the X sensor is a_x , the output x_{sense} of the sensor will be,

$$x_{\text{sense}} = g_x a_x + b_x . \quad (1)$$

Lastly, the orientation of each sensor may not be perfectly orthogonal to the other two sensors' orientations. The unit vectors \mathbf{n}_x , \mathbf{n}_y and \mathbf{n}_z represent the orientations of the sensors for the X , Y and Z axes. Because of non-orthogonality, \mathbf{n}_z may not align with the direction $\mathbf{n}_{x \times y} = \mathbf{n}_x \times \mathbf{n}_y$ orthogonal to \mathbf{n}_x and \mathbf{n}_y . Non-orthogonality is measured by the angle,

$$\cos^{-1} \frac{\mathbf{n}_z \cdot \mathbf{n}_{x \times y}}{\|\mathbf{n}_z\| \|\mathbf{n}_{x \times y}\|} . \quad (2)$$

These three error components can be corrected by using calibration to find the affine transformation between measurements $\mathbf{x}_{\text{sense}} = [x_{\text{sense}}, y_{\text{sense}}, z_{\text{sense}}]^T$ from the X , Y and Z sensors and an estimate $\mathbf{x}_{\text{cal}} = [x_{\text{cal}}, y_{\text{cal}}, z_{\text{cal}}]^T$ of the components of the true acceleration vector in orthogonal X , Y and Z coordinates. The affine transformation for calibration

is characterized by the matrix \mathbf{K} and offset vector $\mathbf{x}_{\text{offset}}$

$$\mathbf{x}_{\text{cal}} = \mathbf{K} \mathbf{x}_{\text{sense}} + \mathbf{x}_{\text{offset}} . \quad (3)$$

To get the intuition for how calibration corrects the three types of errors (3) is manipulated to show the estimated characteristics for the X , Y and Z sensors. The inverse matrix \mathbf{K}^{-1} is applied to both sides and we solve for $\mathbf{x}_{\text{sense}}$ to get

$$\mathbf{K}^{-1} \mathbf{x}_{\text{cal}} - \mathbf{K}^{-1} \mathbf{x}_{\text{offset}} = \mathbf{x}_{\text{sense}} . \quad (4)$$

Comparing (4) to (1), it can be seen that the estimated orientation of the sensors as plotted in Fig. 1 are the rows of the inverse matrix \mathbf{K}^{-1} . For example, the orientation of the X axis sensor is the direction for \mathbf{x}_{cal} that produces the largest measured offset-corrected response in x_{sense} . While the direction of each row of \mathbf{K}^{-1} represents the sensor orientation, the l^2 norm [13] of each row is the gain for each sensor,

$$\mathbf{K}^{-1} = \begin{bmatrix} g_x & 0 & 0 \\ 0 & g_y & 0 \\ 0 & 0 & g_z \end{bmatrix} \begin{bmatrix} \mathbf{n}_x^T \\ \mathbf{n}_y^T \\ \mathbf{n}_z^T \end{bmatrix} . \quad (5)$$

Furthermore, the term $-\mathbf{K}^{-1} \mathbf{x}_{\text{offset}}$ in (4) is the offset for each sensor. The offsets are also plotted in Fig. 1 and represent the acceleration vectors that calibration estimates would produce a reading of 0g on each sensor.

3. AUTOMATIC CALIBRATION METHOD

The proposed automatic calibration method relies only on the data recorded during durations when the sensor happens to be at rest. The key intuition is that while the sensor is at rest the only observed acceleration is the constant acceleration of $1g \approx 9.81 \text{ m/s}^2$ due to gravity. Deviations in the magnitude of the sensor output measurements from $1g$ can be attributed to calibration errors and can be used to recover the calibration matrix \mathbf{K} and offset $\mathbf{x}_{\text{offset}}$.

Automatic calibration first separates out the segments of data for which the sensor is at rest. The observed data is now the time varying vector $\mathbf{x}_{\text{sense}}(t) = [x_{\text{sense}}(t), y_{\text{sense}}(t), z_{\text{sense}}(t)]$ which is segmented into intervals $k = 1, 2, \dots, N$ each of duration T samples with the k^{th} segment denoted $\mathbf{x}_{\text{sense},k}(t)$. For each interval that the variance $\text{Var}(k) = \sum_{t=1}^T (\|\mathbf{x}_{\text{sense},k}(t)\| - \sum_{t'=1}^T \|\mathbf{x}_{\text{sense},k}(t')\|/T)^2 / (T-1)$ of the acceleration's magnitude is sufficiently small, the interval is added to the data set used for calibration,

$$\mathcal{K}_{\text{rest}} = \{k \in 1, 2, \dots, N \mid \text{Var}(k) < \tau\} . \quad (6)$$

For each interval in $\mathcal{K}_{\text{rest}}$, calibration calculates the mean acceleration vector $\bar{\mathbf{x}}_{\text{sense},k} = \sum_{t=1}^T \mathbf{x}_{\text{sense},k}(t)/T$. Figure 2 shows that the plotted the mean acceleration vectors from the set $\mathcal{K}_{\text{rest}}$ lie on the surface of an ellipsoid. In the absence of calibration errors the resting data would lie on a sphere of

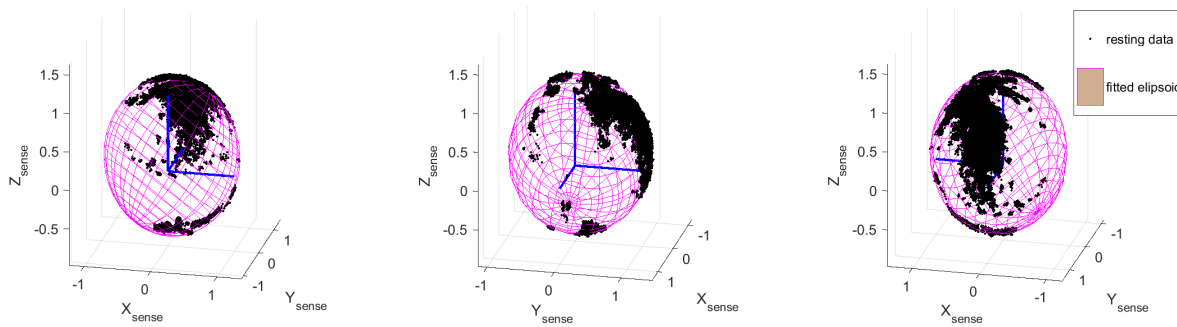


Fig. 2. Data used for automatic calibration and the fitted ellipsoid.

radius 1g centered at the origin, while errors due calibration cause the surface to be an ellipsoid. Figure 2 shows that an ellipsoid is a good fit for the resting data and all sources of error other than calibration are small and safely modeled as zero mean additive white Gaussian noise (AWGN). Assuming AWGN errors, the parameters of the ellipsoid can be found by minimizing the sum of squared distances between the set of resting accelerations and the surface of the ellipsoid. The equation of the resulting fitted ellipsoid is,

$$(\mathbf{x}_{\text{sense}} - \mathbf{b})^T \mathbf{Q} (\mathbf{x}_{\text{sense}} - \mathbf{b}) = 1, \quad (7)$$

where \mathbf{Q} is a positive definite matrix and \mathbf{b} is a vector of the coordinates for the center. The matrix \mathbf{Q} and center \mathbf{b} are determined by using a nonlinear least-squares curve fitting [14, 15] (e.g. such as `lsqnonlin` in *Matlab*). Using the Cholesky factorization of $\mathbf{Q} = \mathbf{K}^T \mathbf{K}$ we get the matrix \mathbf{K} needed to produce the calibrated accelerometer data $\mathbf{x}_{\text{cal}} = \mathbf{K} \mathbf{x}_{\text{sense}} + \mathbf{x}_{\text{offset}}$ lying on a sphere,

$$\begin{aligned} (\mathbf{x}_{\text{sense}} - \mathbf{b})^T \mathbf{K}^T \mathbf{K} (\mathbf{x}_{\text{sense}} - \mathbf{b}) &= 1 \\ (\mathbf{K} (\mathbf{x}_{\text{sense}} - \mathbf{b}))^T \mathbf{K} (\mathbf{x}_{\text{sense}} - \mathbf{b}) &= 1 \\ \mathbf{x}_{\text{cal}}^T \mathbf{x}_{\text{cal}} &= 1 \end{aligned} \quad (8)$$

where (8) is the equation for a sphere on which the calibrated resting data points lie on. From (8) we can see that the rotation and gain matrix for calibration is \mathbf{K} and the offset vector in (3) is $\mathbf{x}_{\text{offset}} = -\mathbf{K} \mathbf{b}$.

Note that the absolute orientation of the sensor is not known at any time within the duration of observed data. This means that the absolute rotation of the three axis accelerometers remains a free parameter. By choosing an upper triangular Cholesky factorization it is assumed that sensor for the Z axes is oriented correctly ($\mathbf{n}_z = [0 \ 0 \ 1]^T$), and only the relative calibration errors are corrected. Results show that the most significant calibration errors are due to offset and this relative calibration provides high accuracy.

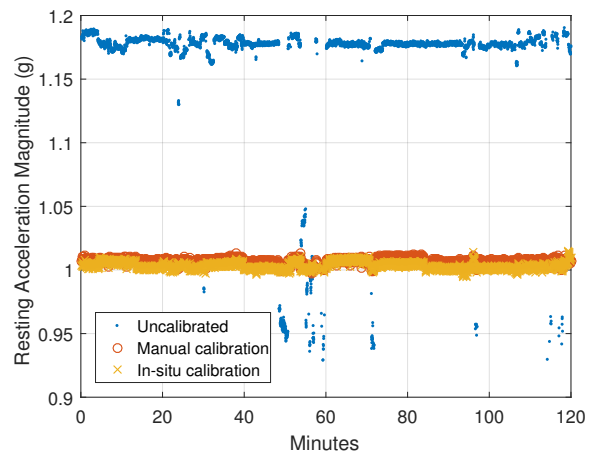


Fig. 3. Validation of calibration for resting data from a different day than used for calibration.

4. RESULTS

To validate the proposed technique, calibration parameters are first found for the three-axis accelerometers in the five devices given in Table 1. The three-axis accelerometers are integrated into *BioStampRC*[®] devices provided by *MC10, Inc.*. Acceleration data is collected at 50 Hz and in-situ calibration looks for 1 second durations ($T = 50$ samples) of rest to isolate the acceleration of gravity. A threshold of $\tau = 10^{-4} \text{g}^2$ is used to find the resting intervals. While T and τ are heuristically chosen, Fig. 2 shows that the chosen parameters both limit the noise and outliers present in the data, and provide a sufficient number of data points for fitting.

In addition to proposed in-situ calibration using the parameters of the fitted ellipsoid, a second *manual* calibration [16] is also performed for reference. The manual calibration takes advantage of an offline procedure to calibrate the same five devices as the proposed calibration. The offline procedure

Table 1. Comparison of the calibration corrections calculated by both the manual and in-situ techniques.

Device ID	1		2		3		4		5	
	manual	in-situ	manual	in-situ	manual	in-situ	manual	in-situ	manual	in-situ
X offset	+0.04g	+0.03g	-0.01g	-0.01g	+0.09g	+0.09g	+0.05g	+0.05g	+0.03g	+0.02g
Y offset	+0.07g	+0.08g	+0.01g	+0.02g	-0.00g	-0.01g	+0.06g	+0.07g	-0.00g	+0.01g
Z offset	+0.17g	+0.17g	+0.15g	+0.15g	+0.19g	+0.20g	+0.19g	+0.18g	+0.20g	+0.20g
X gain	1.01	1.01	1.00	1.00	0.96	0.96	1.01	1.00	0.99	0.99
Y gain	1.05	1.04	1.05	1.04	1.04	1.04	1.02	1.02	1.05	1.05
Z gain	0.99	0.99	0.98	0.98	1.01	1.00	0.97	0.97	0.98	0.99
X non-ortho	2.1°	2.8°	2.5°	3.2°	2.7°	3.6°	2.3°	3.2°	2.2°	2.9°
Y non-ortho	2.0°	2.7°	2.5°	3.2°	2.6°	3.5°	2.2°	3.2°	2.1°	2.8°
Z non-ortho	0.6°	1.0°	0.6°	0.7°	0.9°	1.2°	0.6°	0.2°	0.8°	0.7°

requires placing the devices in six known orientations:

$$\mathbf{x}_{\text{cal}} \in \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \right\}; \quad (9)$$

and records the response of the X , Y and Z axes accelerometer sensors for each orientation. Because the orientations are known, the calibration can be computed from the observed response using a least squares solution. A key distinction of the manual calibration is that offline procedure provides an absolute frame of reference for the sensor orientations. The proposed in-situ calibration does not use any offline procedures where the sensor's orientation is known by any means other than the observed readings due to gravity. Thus the in-situ calibration can only correct the three types of errors given in Table 1, and allows the overall orientation of the three-axis sensor within the device to be a free parameter. Table 1 shows that even without absolute knowledge of the device orientation, the in-situ calibration recovers nearly the same calibration parameters as the manual procedure.

The performance of the both the manual and in-situ procedures are tested using a set of acceleration data recorded on a different day than the data used for ellipsoid fitting or manual calibration. For this separately recorded data the resting durations are extracted in the same manner as (6) and the calibration parameters calculated by both methods are applied to validate which of these provides a better estimate of the expected 1g acceleration due to gravity. Without calibration, the magnitude varies between .81g and 1.30g for an root mean square error (RMSE) of .13g. The manual calibration reduces the variation to between .98g and 1.03g for a RMSE of .01g, while the in-situ calibration estimate varies between .97g and 1.06g for a RMSE of .04g. These results demonstrate that the proposed in-situ calibration approach provides comparable or better accuracy than the offline manual calibration method.

5. CONCLUSION

This paper has proposed a method of calibrating three-axis accelerometer devices without needing to perform any specific calibration procedures or use known reference orientations.

Calibration relies on detecting when the sensor is at rest to isolate durations when the only acceleration acting on the device is the known acceleration due to gravity. As compared to a prior manual calibration procedure that utilized known reference orientations, the proposed in-situ calibration provides comparable accuracy. The main advantage is the ability to compute the calibration parameters from an arbitrary set of recorded data without needing to access the sensor device. The only requirement is that the recorded data include sufficient durations of resting data, which is readily met in our primary target application of body-worn health sensing.

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