A Novel Scheme for Support Identification and Iterative Sampling of Bandlimited Graph Signals

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BACKGROUND

- Graph Signal Processing
  - Modeling network processes by exploiting the underlying graph structures
  - Applications: sensor and social networks, transportation systems, gene regulatory networks
- Sampling and Reconstruction
  - Selecting a small representative subset of graph nodes
  - Applications: constrained sensing in sensor networks, data summarization

APPLICATIONS:

- Support recovery error vs. various number of effective noise power
- Success rate (invertibility of CU)

PERFORMANCE ANALYSIS

- Proposed scheme guarantees perfect recovery in noiseless case for all connected graphs with general structures and with normal adjacency.

Theorem 1:
Let \( \mathcal{S} \) be the sampling set constructed by Algorithm 1 and let \( \mathcal{C} \) be the corresponding sampling matrix such that \( |\mathcal{S}| = k \). Then, the matrix \( \mathcal{C} \mathcal{U} \) is invertible.

- Proof's remarks:
  - An inductive argument
  - Iterative selection of linearly independent \( u_i \)'s
  - Zero residual norm only after the last iteration

Sampling under bounded noise
- Assumption: \( |\mathcal{N}| \leq \epsilon_0 \)
- Explicit bound on reconstruction error

\[
|\hat{x} - x| \leq \sigma_{\text{max}}(U_{\mathcal{S}}^T Q_S^{-1} U_{\mathcal{S}}) \epsilon_0
\]

- Guaranteed existence of inverse matrices under Algorithm 1
- Preserving statistical characteristics (e.g. whiteness) of effective noise

SUPPORT IDENTIFICATION

- Support recovery given \( P \) historical templates \( X = [x^1, \ldots, x^p] \in \mathbb{R}^{N \times p} \) with shared support from noisy observations \( Y = X + N \)
- Equivalent task: estimating sparse GFTs \( \bar{X} = [\bar{x}^1, \ldots, \bar{x}^p] = V^T X \)
- Proposed optimization based on block sparsity of \( X \)

\[
\min_{\bar{X}} \| \bar{X} - V^T Y \|_F^2 \quad \text{s.t.} \quad \| \bar{X} \|_{2,0} \leq k
\]

- Closed-form solution via row-wise \( l_2 \) norm thresholding on \( Y = V^T Y \)

\[
\bar{X}(i,:) = \begin{cases} \bar{Y}(i,:) & \| \bar{Y}(i,:) \|_2 \geq k^{th} \text{ largest } l_2 \text{ norm o.w.} \\ 0 \end{cases}
\]

Theorem 2
Assume \( Y \) is orthogonal. Under bounded noise assumption, the GFTs \( \bar{X} \) and support \( \mathcal{K} \) are identifiable if

\[
\min_{i \in \mathcal{K}} \| \bar{X}(i,:) \|_2 > 2\epsilon_0 \sqrt{P}.
\]

- Perfect support identification in the absence of noise with \( P = 1 \)
- Easier satisfiability of the established sufficient condition for larger \( P \)

CONCLUSION

- Our contributions:
  - Iterative sampling of graph signals in non-Bayesian setting with performance guarantees
  - Extension to unknown support scenario: support identification from historical templates of the graph signal
- Future work: Joint support identification and sampling