

# On the Relationship of Dielectrophoresis and Electrowetting

Thomas B. Jones\*

*Department of Electrical & Computer Engineering, University of Rochester,  
Rochester, New York 14627*

*Received February 8, 2002. In Final Form: March 18, 2002*

Electrostatic fields have two observable effects upon the hydrostatics of liquids. These are (i) a net force exerted on the liquid if the electric field is nonuniform and (ii) a change in the contact angle. For the case of electrodes coated with thin layers of dielectric material, both of these effects are most certainly electromechanical in nature. Though often seen to occur together, the bulk electromechanical and electrowetting effects are distinct and can be expected to occur independently under proper conditions. A derivation using the methods of lumped parameter electromechanics demonstrates that the net force on a liquid mass situated between two parallel, coated electrodes can be determined with no reference to the actual shape of the meniscus. By the very nature of its derivation, this electrostatic force cannot be localized as to where it acts on the liquid. The conclusion is that the term “electrowetting” should be restricted in its use to denote the effect of the electric field upon the contact angle. On the other hand, translational forces harnessed in certain microfluidic applications are better referred to as examples of the net electromechanical force resulting from electric field nonuniformity. The high-frequency limit of the electromechanical response, equivalent to the case of an insulative liquid, is recognized as liquid dielectrophoresis.

## Introduction

When a droplet of conducting liquid is placed atop a horizontal metallic electrode that has been covered by a thin dielectric layer, a voltage applied between the electrode and the droplet will result in an observable reduction of the contact angle the liquid makes with the solid surface. This is an example of the so-called electrowetting effect. It is also known that a *net* electromechanical force can be exerted on a liquid mass by applying voltage to a proper arrangement of similarly coated electrodes. This phenomenon is sometimes grouped together with electrowetting. Though both are electromechanical in nature, they are really not the same. One influences the wetting of the solid surface by the liquid, and the other makes it possible to transport the liquid. Experiments may be devised to provide independent demonstration of either electrowetting or the electromechanical force effect.

In this paper, we invoke the standard methods of lumped parameter electromechanics to determine the net force of electrical origin on a liquid mass located between two parallel, dielectric-coated electrodes. First, the system capacitance is identified, and then a derivative of an energy function is taken with respect to a translational mechanical variable. These derivations, verified by surface integrals of the Maxwell stress tensor, make no assumptions whatsoever about the profile of the liquid meniscus or the contact angle. They likewise make no representation about the localization of the electrical force acting on the liquid. Conducting and insulating liquids are treated separately. The conducting case, being the more familiar, is closely related to work published in the last 15 years on electrowetting. The insulating case, a simple variant of the standard textbook example of the ponderomotive force, in fact describes the high-frequency, dielectrophoretic (DEP) actuation of semi-insulative, polar liquids such as water.

The argument presented in this paper leads to a conclusion that the electromechanical force effect, which

fully explains all observable bulk motions of liquid masses (including the height-of-rise effect and microfluidic droplet actuation), should be distinguished from electrowetting. Restricting the use of the term “electrowetting” to describe the observable influence of an electric field on contact angle seems advisable. At the same time, an assuring consistency of the generally accepted electrowetting theory with the predictions of the electromechanical force model is readily shown by combining Laplace’s capillary equation with the electrowetting equation to predict the liquid height-of-rise. This consistency is shown for both conductive and insulative liquids.

## Background

It has been known for over a century that a nonuniform electric field can dramatically influence the hydrostatic equilibrium of a dielectric liquid. In his classic experiment, Pellat demonstrated that an insulating dielectric liquid rises upward against the pull of gravity when a voltage is applied between parallel electrodes.<sup>1</sup> Figure 1 shows the basic arrangements of the original experiment. If the experiment is performed in air and if the liquid has mass density equal to  $\rho_1$  and dielectric constant equal to  $\kappa_1$ , then the well-known expression for the dielectric height-of-rise can be used.<sup>2</sup>

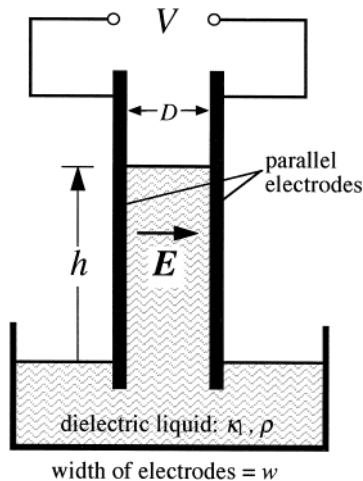
$$h \approx \frac{(\kappa_1 - 1)\epsilon_0 E^2}{2\rho_1 g} \quad (1)$$

where  $E$  is the electric field between the electrodes and  $g = 9.81 \text{ m/s}^2$  is the terrestrial gravitational acceleration. Equation 1 is accurate if the plate spacing  $D$  is small enough so that the uniform field approximation,  $E = V/D$ ,

(1) Pellat, H. Mesure de la force agissant sur les diélectriques liquides non électrisés placés dans un champ élitrique. *C. R. Acad. Sci. Paris* **1895**, *119*, 691–694.

(2) Jones, T. B.; Melcher, J. R. Dynamics of electromechanical flow structures. *Phys. Fluids* **1973**, *16*, 393–400. Jones, T. B. Hydrostatics and steady dynamics of spatially varying electromechanical flow structures. *J. Appl. Phys.* **1974**, *45*, 1487–1491.

\* E-mail: jones@ece.rochester.edu.



**Figure 1.** Pellat's original experiment demonstrating the height-of-rise of an insulating, dielectric liquid using parallel electrodes.

is applicable. At one time, the U.S. space program explored various ways to utilize this force for the management and control of propellants and other liquids in zero gravity applications.<sup>2-4</sup> Melcher was probably first to refer to the phenomenon as liquid *dielectrophoresis*. In doing so, he was appropriating a term originally coined by Pohl to describe the attraction of small dielectric particles into regions of relatively stronger electric fields.<sup>5</sup> The choice of this word to describe the analogous behavior of liquids subjected to nonuniform electric fields was motivated by recognition of the common nature of the physical mechanism exemplified by both Pellat's original experiment with dielectric liquids and Pohl's later work with dielectric particles, namely, the ponderomotive force. The net electrical body force acting on liquids is usually formulated in terms of the Korteweg–Helmholtz body force density.<sup>6</sup>

$$\vec{f} = \rho_f \vec{E} - \frac{1}{2} E^2 \nabla \epsilon + \nabla \left[ \frac{1}{2} E^2 \frac{\partial \epsilon}{\partial \rho} \right] \quad (2)$$

where  $\rho_f$  is the density of free electric charge within the liquid. In an incompressible liquid, we may ignore any influence of electrostriction, that is, the third term in eq 2, on the hydrostatics. Also, it is assumed that  $\rho_f = 0$ . Then, in the case of Pellat's experiment with a homogeneous liquid, eq 2 places a singular, normal force right at the free surface of the liquid between the electrodes. Such a placement does not make sense on physical grounds. But the correct usage of eq 2 is to calculate the *total* force on a ponderable body by an appropriate volume integral, not to pinpoint the distribution of electrical body forces acting within it. (This point takes on great importance when we try to sort out the matter of how an electric field influences capillary phenomena.) In trying to explain how the liquid is lifted up by the electric field, it is helpful to think of the dipole force acting on the polarized molecules of the liquid in the fringing field region at the bottom of the electrodes. This force can be expressed as  $\vec{P} \cdot \nabla \vec{E}$ , where  $\vec{P}$  is the vector density of dipoles in the liquid,<sup>7</sup> and it is nonzero only where the electric field is nonuniform.

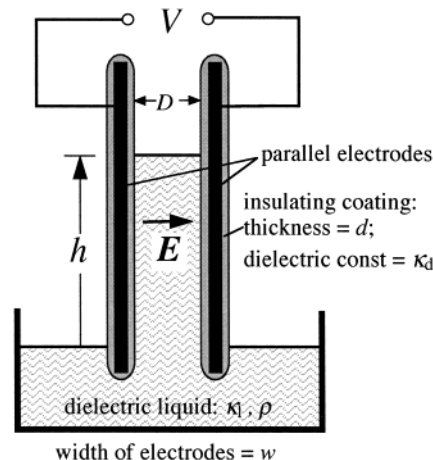
(3) Melcher, J. R.; Hurwitz, M.; Fax, R. *Dielectric liquid expulsion. J. Spacecr. Rockets* **1969**, *6*, 961–967.

(4) Jones, T. B.; Perry, M. P. Electrohydrodynamic heat pipe experiments. *J. Appl. Phys.* **1974**, *45*, 2129–2132.

(5) Pohl, H. A. The motion and precipitation of suspensoids in divergent electric fields. *J. Appl. Phys.* **1951**, *22*, 869–871.

(6) Landau, L. D.; Lifshitz, E. M. *Electrodynamics of continuous media*; Pergamon: Oxford, 1960; Section 15.

(7) Becker, R. *Electromagnetic fields and interactions*; Dover: New York, 1982; Section 35.

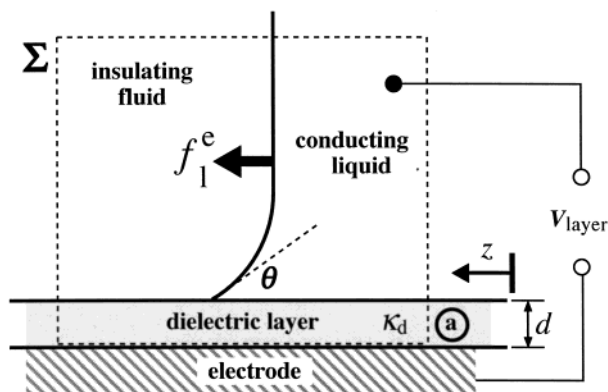


**Figure 2.** Modification of Pellat's experiment for a conductive liquid using electrodes coated with an insulating dielectric layer. Note that the liquid is not grounded.

In his experiments, Pellat used insulating dielectric liquids. If a conductive liquid is used instead, a similar height-of-rise phenomenon is to be expected and eq 1 is still applicable; however, at least in a terrestrial gravitational field, rapid Joule heating sets in at a field strength well below the value required to achieve a significant liquid elevation. Even distilled or deionized water, which has conductivity  $\sigma \approx 10^{-4}$  S/m (after exposure to air), is apt to boil before a 1 cm height-of-rise can be achieved. Fortunately for applications on the micro-scale, the heating problem is ameliorated to a significant extent by physical scaling laws, which simultaneously favor electrostatic forces and promote conductive heat dissipation in small structures. In other words, heating can be more successfully dealt with, or at least tolerated, for devices having dimensions of <100 microns. Therefore, the rapidly growing interest in electric-field-mediated microfluidic devices for the laboratory on a chip suggests that renewed consideration be given to the exploitation of dielectrophoresis for aqueous media. Another reason for serious reconsideration of dielectrophoresis as a way to manipulate liquids such as water is to better expose its relationship to the electrowetting phenomenon.

**Pellat's Experiment for a Conductive Liquid.** To achieve the height-of-rise phenomenon with a conductive liquid, it is necessary to coat the electrodes with a thin dielectric layer to eliminate electrolysis. Refer to Figure 2, showing two parallel electrodes at spacing  $D$ , coated with an insulating dielectric of thickness  $d \ll D$  and dielectric constant  $\kappa_d$  and partially immersed in a conductive liquid. For this situation, a new expression must be found for  $h$ , which we refer to more generally as the electromechanical height-of-rise. In the absence of surface tension, the determination of  $h$  may be done using either lumped parameter electromechanics or the Maxwell stress tensor. A general approach is to consider the geometry of Figure 3 and to determine  $f_1^e$ , the  $z$ -directed force of electrical origin per unit length of the contact line. The contact line is the line traced on the dielectric coating by the triple junction where solid, liquid, and gas come together.

Application of the method of virtual displacement from lumped parameter electromechanics provides us a means to calculate the force of electrical origin acting on the



**Figure 3.** Geometry used to calculate the force of electrical origin per unit length of the contact line of a conducting liquid adjacent to a coated electrode. The dashed line defines a closed surface  $\Sigma$  used for the Maxwell stress integration in Appendix A. Note that no precise location for the electrical force is specified.

liquid.<sup>8</sup> The starting point for this method is an expression for the system capacitance per unit length.

$$c(z) = \frac{\kappa_d \epsilon_0 z}{d} + \text{constant} \quad (3)$$

Using standard methods, the coenergy per unit length  $w_e'$  is then

$$w_e' = c(z) V_{\text{layer}}^2 / 2 \quad (4)$$

and the net force of electrical origin, also per unit length of the contact line, is equal to the partial derivative of the coenergy with respect to  $z$ .

$$f_1^e = \frac{\partial w_e'}{\partial z} = \frac{V_{\text{layer}}^2}{2} \frac{dc}{dz} = \frac{\kappa_d \epsilon_0 V_{\text{layer}}^2}{2d} \quad (5)$$

This force pulls the liquid surface from right to left as shown in Figure 3. Refer to Appendix A for an alternative derivation of eq 5 based on the Maxwell stress tensor.

Because one can easily envision concentrated electric field lines exerting a Coulombic force on the free charge induced at the surface of the liquid near the contact line, it is tempting to attribute this electrical force to the contact line itself. But to do so is misleading and, furthermore, quite unnecessary if the goal is to predict the observable net force exerted on the liquid by the electric field. Notice that eq 5 for the upward force is not dependent on the shape of the liquid meniscus, because, as long as  $d \ll D$ , the capacitance expression  $c(z)$  itself is sensitive only to the location of the contact line. For exactly the same reason that eq 2 is not really intended to represent the true distribution of the electrical body force, it is not necessary to localize the force  $f_1^e$  at the contact or anywhere else.

To determine the electromechanical height-of-rise, we set the total  $z$ -directed (vertical) force of electrical origin equal to the net gravitation head that must be balanced to hold up the liquid column.

$$f_1^e \times 2w = \rho_1 g h w D \quad (6)$$

where  $w$  is the width of the electrodes. Combining eqs 5 and 6 and defining the total voltage applied

to the parallel electrode structure to be  $V = 2V_{\text{layer}}$ , one obtains

$$h = \frac{\kappa_d \epsilon_0 V^2}{4\rho_1 g d D} \quad (7)$$

This expression is identical to that of Welters and Fokkink.<sup>9</sup>

**Electrostatic Effects on Capillarity.** In the above, we have completely neglected capillary behavior, treating the height-of-rise phenomenon in conductive liquids as a problem of classical electromechanics. This approach is adequate on the scale of centimeters, but in any experiment that might be conducted with liquids, especially water, on the scale of millimeters or smaller, we must anticipate capillary behavior and particularly the well-known influence of an electric field on the solid/liquid contact angle  $\theta$ . Berge<sup>10</sup> and then others<sup>9,11–20</sup> have investigated the electrowetting effect in depth, and interesting potential microfluidic systems based on exploitation of the phenomena have been identified.<sup>21–23</sup>

Consider then the modified Pellat experiment using a conductive liquid, but now with the influence of capillarity on the hydrostatic equilibrium taken into account. As shown in Figure 4, the liquid rises to a height  $h'$  by the normal capillary effect and then an additional amount  $\Delta h$  when the voltage  $V$  is applied. Meniscus heights, being very small, are neglected here. From the Laplace equation, we have

$$h' = \frac{2\gamma_{lv} \cos(\theta_0)}{\rho g D} \quad (8)$$

where  $\theta_0$  is the contact angle with no applied voltage and  $\gamma_{lv}$  is the surface tension at the liquid/vapor interface. When the voltage is applied, the contact angle changes to  $\theta_E$ . From Berge<sup>9</sup> and others,  $\theta_E$  is related to the initial

(9) Welters, W. J. J.; Fokkink, L. G. J. Fast electrically switchable capillary effects. *Langmuir* **1998**, *14*, 1535–1538.

(10) Berge, B. Électrocapillarité et mouillage de films isolants par l'eau. *C. R. Acad. Sci., Ser. II* **1993**, 157–163.

(11) Vallet, M.; Berge, B.; Vovelle, L. Electrowetting of water and aqueous solutions on poly(ethylene terephthalate) insulating films. *Polymer* **1996**, *12*, 2465–2470.

(12) Verheijen, H. J. J.; Prins, M. W. J. Reversible electrowetting and trapping of charge: model and experiments. *Langmuir* **1999**, *15*, 6616–6620.

(13) Vallet, M.; Vallade, M.; Berge, B. Limiting phenomena for the spreading of water on polymer films by electrowetting. *Eur. Phys. J.* **1999**, *B11*, 583–591.

(14) Blake, T. D.; Clarke, A.; Stattersfield, E. H. An investigation of electrostatic assist in dynamic wetting. *Langmuir* **2000**, *16*, 2928–2935.

(15) Peykov, V.; Quinn, A.; Ralston, J. Electrowetting: a model for contact-angle saturation. *Colloid Polym. Sci.* **2000**, *278*, 789–793.

(16) Digilov, R. Charge-induced modification of contact angle: The secondary electrocapillary effect. *Langmuir* **2000**, *16*, 6719–6723.

(17) Decamps, C.; De Coninck, J. Dynamics of spontaneous spreading under electrowetting conditions. *Langmuir* **2000**, *16*, 10150–10153.

(18) Rosslee, C.; Abbott, N. Active control of interfacial wetting. *Curr. Opin. Colloid Interface Sci.* **2000**, *5*, 81–87.

(19) Janocha, B.; Bauser, H.; Oehr, C.; Brunner, H.; Göpel, W. Competitive electrowetting of polymer surfaces by water and decane. *Langmuir* **2000**, *16*, 3349–3354.

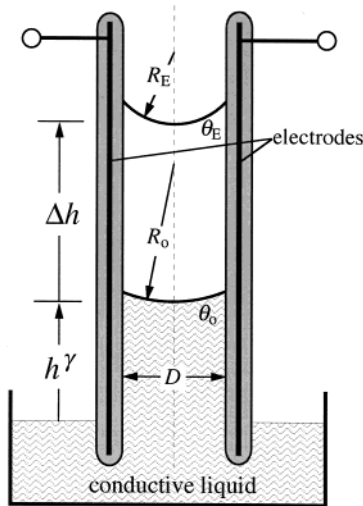
(20) Quilliet, C.; Berge, B. Electrowetting: a recent breakout. *Curr. Opin. Colloid Interface Sci.* **2001**, *6*, 34–39.

(21) Pollack, M. G.; Fair, R. B.; Shenderov, A. D. Electrowetting-based actuation of liquid droplets for microfluidic actuation. *Appl. Phys. Lett.* **2000**, *77*, 1725–1726.

(22) Prins, M. W. J.; Welters, W. J. J.; Weekamp, J. W. Fluid control in multichannel structures by electrocapillary pressure. *Science* **2001**, *291*, 277–280.

(23) Lee, J.; Moon, H.; Fowler, J.; Schoellhammer, T.; Kim, C.-J. Electrowetting and electrowetting-on-dielectric for microscale liquid handling. *Sens. Actuators* **2002**, *95*, 259–268.

(8) Woodson, H. H.; Melcher, J. R. *Electromechanical dynamics, part I. Discrete systems*; Wiley: New York, 1968; Chapter 3.



**Figure 4.** Depiction of the combined effects of capillarity and electric field induced DEP actuation. Note that the radius of curvature of the meniscus decreases as voltage  $V$  is increased.

contact angle, the voltage, and the properties of the dielectric layer by an electrowetting equation.

$$\cos(\theta_E) - \cos(\theta_0) = \frac{\kappa_d \epsilon_0 V_{\text{layer}}^2}{2d\gamma_{\text{lv}}} \quad (9)$$

Note that  $\theta_E < \theta_0$ . Equation 9 is predicated on the assumption that the liquid surface tension  $\gamma_{\text{lv}}$  is not dependent on the electrostatic field.<sup>10,12,14,16</sup> Using eqs 8 and 9, an expression for  $h$ , the total height-of-rise of the liquid between the electrodes, is now postulated in terms of the interfacial tension and the voltage-altered contact angle.

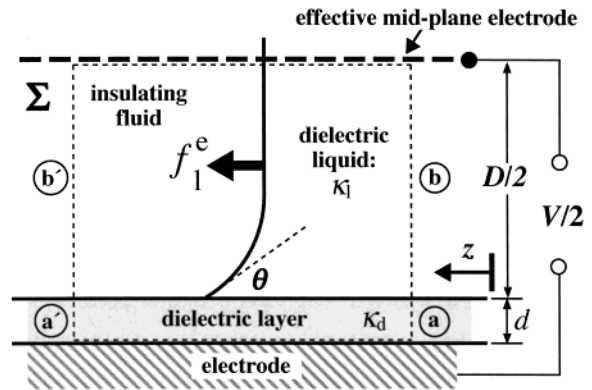
$$h = \frac{2\gamma_{\text{lv}} \cos(\theta_E)}{\rho g D} = h^\gamma + \frac{\kappa_d \epsilon_0 V^2}{4\rho g d D} \quad (10)$$

One can see that the second, voltage-dependent term in eq 10 is identical to the electromechanical height-of-rise expression, eq 7. Thus, the electrowetting model and the lumped parameter electromechanical model (which completely ignores capillarity) both predict the same voltage-induced height-of-rise. The only contribution to the net response of the liquid that is directly and unambiguously attributable to electrowetting, that is, not successfully predicted by the electromechanical model, is the change in curvature of the liquid surface, which slightly increases the vertical reach of the meniscus. The magnitude of this effect, readily calculated, is ordinarily quite small for practical microfluidic applications and can be neglected.

**Frequency Influence on Electromechanical Height-of-Rise.** If the conductive liquid has finite electrical conductivity  $\sigma_1$  and dielectric constant  $\kappa_1$ , then there will exist a critical frequency  $f_c$  for ac voltage excitation marking the transition from conductive to dielectric behavior. From elementary circuit considerations, this frequency is

$$f_c = \frac{\sigma_1}{2\pi\epsilon_0[\kappa_1 + \kappa_d D/2d]} \quad (11)$$

For  $f \gg f_c$ , the liquid behaves like a dielectric. Ohmic current is dominated by the displacement current, which means that the liquid volume is no longer an equipotential. The electric field penetrates the liquid so the net trans-



**Figure 5.** Geometry used to calculate force of electrical origin per unit length of the contact line for a dielectric liquid adjacent to a coated electrode. The heavy dashed line represents a virtual electrode located at the midplane of the electrode structure. The box defined by the lighter dashed line defines the closed surface  $\Sigma$  used for the Maxwell stress integration in Appendix A.

lational force on the liquid is reduced. Furthermore, the electric field can have a significant influence on the shape taken by the liquid. To calculate the net force on the liquid in this high-frequency regime, a new lumped parameter (capacitive) circuit model shown in Figure 5 must be used. Note that this circuit actually represents one-half of the structure, so that the midplane becomes a virtual electrode, the voltage is  $V/2$ , and the thickness of the liquid layer is  $D/2$ . The capacitance per unit length is

$$c(z) = \frac{\epsilon_0 z}{d/\kappa_d + D/2\kappa_1} + \frac{\epsilon_0(L-z)}{d/\kappa_d + D/2} + \text{constant} \quad (12)$$

In eq 12, the transition region close to the interface contributes only a constant term to the capacitance. Because all constant terms disappear when the derivative is taken, they contribute nothing to the force of electrical origin. Once again defining coenergy and taking its derivative with respect to  $z$ , the force per unit length of the contact line is

$$f_1^e = \frac{\epsilon_0 V^2}{8} \left[ \frac{1}{d/\kappa_d + D/2\kappa_1} - \frac{1}{d/\kappa_d + D/2} \right] \quad (13)$$

Refer to Appendix A which uses the Maxwell stress tensor to confirm this result. When this expression is substituted into eq 6, the dielectric height-of-rise is

$$\Delta h = \frac{\epsilon_0 V^2}{2\rho g D} \left[ \frac{1}{2d/\kappa_d + D/\kappa_1} - \frac{1}{2d/\kappa_d + D} \right] \quad (14)$$

In the appropriate limits, this equation reduces to eq 1 or eq 7. Because  $D \gg d$  and  $\kappa_1 \gg \kappa_d$  (for the important case of water), eq 14 simplifies.

$$\Delta h \approx \frac{\epsilon_0 V^2}{2\rho g D [2d/\kappa_d + D/\kappa_1]} \quad (15)$$

Using the electrowetting model to calculate  $f_1^e$  for the dielectric liquid case requires the replacement of eq 9 by an expression accounting for the applied voltage between

the dielectric layer and the liquid. Generalizing an expression of Welters and Fokkink,<sup>9</sup> one may write

$$\cos(\theta_E) - \cos(\theta_0) = \frac{1}{\gamma_{vl}} \int_0^V \sigma_f(V) dV \quad (16)$$

where  $\sigma_f(V)$  is the free electric surface charge density on the metal electrode below the dielectric layer.

$$\sigma(V) = \kappa_d \epsilon_0 E_d(V) \quad (17)$$

In eq 17,  $E_d$  is the uniform electric field in the dielectric layer.

$$E_d(V) = \frac{V/2\kappa_d}{[d/\kappa_d + D/2\kappa_d]} \quad (18)$$

When the integration of eq 16 is performed, the result is

$$\cos(\theta_E) - \cos(\theta_0) = \frac{\epsilon_0 V^2}{4\gamma_{vl}[2d/\kappa_d + D/\kappa_1]} \quad (19)$$

Then, according to the electrowetting model, the total height-of-rise is

$$h = \frac{2\gamma_{lv} \cos(\theta_E)}{\rho g D} = h' + \frac{\epsilon_0 V^2}{2\rho g D[2d/\kappa_d + D/\kappa_1]} \quad (20)$$

Note that the second term in eq 20 is identical to eq 15, so that once again the electrowetting model is shown to be consistent with lumped parameter analysis and the Maxwell stress tensor.

### Discussion

To appreciate the relationship between the electromechanical height-of-rise and the electrowetting effect, it is instructive to recognize the similarity of the assumptions and methods invoked in their models. First, the two derivations share the common feature that neither requires any detailed knowledge of the locally intensified electric field. This same advantage is often exploited to analyze electromechanical actuators, where knowledge of the fringing fields is not required to calculate the electrical force, even though the origin of this force seems to be the fringing field. Second, both methods use energy arguments and invoke the principle of virtual work. The electromechanical analyses presented here use coenergy, while typical derivations of the electrowetting equation, eqs 9 and 15, use the Helmholtz free energy.<sup>12,14</sup> Neither of these energy methods is capable of pinpointing the origin of the electrostatic force that raises the liquid or changes the contact angle. This vital point is exemplified by the fact that the coenergy method is indifferent to the details of the shape of the liquid surface. Clearly, it is not the change in contact angle itself that causes the liquid to rise; a solid slab between the electrodes will rise upward with no change in its shape.

Within the restrictions imposed by the cosine function itself upon application of eqs 9 and 16, the electrowetting model yields the same prediction for  $\Delta h$  that we get using lumped parameter electromechanics (and the Maxwell stress tensor). This consistency strongly suggests that, at least for water in contact with insulating solids such as polymers, electrowetting can be regarded as an electromechanical effect.

Except in the hypothetical case of perfect wetting, that is,  $\theta_0 = 0$ , electrowetting is an *observable* change in the

contact angle caused by the Coulombic interaction of electric charges induced at the conductive liquid surface with the intensified electric field near the contact line. On the other hand, the electromechanical height-of-rise, also *observable*, is a net translational force effect, the nature of which depends on the frequency of the applied electric field  $f$ . For  $f \ll f_c$ , a strong, local Coulombic force exerts an upward pull on the liquid surface near the contact line; for  $f \gg f_c$ , the volume polarization force  $\bar{P} \cdot \nabla E$  acts upon the dielectric liquid within the fringing field region at the bottom of the electrodes. Others have recognized the need to distinguish electrowetting from the net electromechanical force responsible for the height-of-rise phenomenon.<sup>16,22,24</sup> The present contribution generalizes this perspective by accounting for dielectrophoretic behavior and showing that the generalized electrowetting model, eq 16, successfully predicts the observable height-of-rise for both conductive and insulative (dielectric) liquid media. This success does not qualify electrowetting as the origin of the electromechanical height-of-rise or vice versa; rather, it provides an expected but reassuring consistency.

The standard electrowetting model does have the serious shortcoming that the contact angle cannot be reduced below a value of zero. This problem is evident both from the intuitive standpoint and from the limit imposed by the cosine function itself, viz., that  $|\cos(\theta_E)| \leq 1$ . Either both eqs 9 and 16 are wrong outright or, far more likely, various evidently energy-dissipating phenomena intervene above some threshold value of the voltage to complicate the attractively simple picture of electrowetting as an observable change in the contact angle. On the other hand, the lumped parameter model does not have this shortcoming and, because it does not depend on the fine details of the shape of the meniscus, remains valid.

Recognizing electrowetting as one aspect of the electromechanical response of a conducting or dielectric liquid to a strong electrostatic field is not a diminishment of the importance of the phenomenon. Electrowetting is an *observable* with very important practical applications that range from high-speed film coating to printing technologies. Furthermore, electrowetting may be useful or even essential in overcoming the highly stictive nature of wetting in DEP microactuation.<sup>25</sup> Indeed, its technological importance dictates that the physical nature of electrowetting be properly and unambiguously defined. With such a goal in mind, consider two parallel, dielectric-coated electrodes spaced far enough apart so that  $D \gg h$  as depicted in Figure 6a. With no electrical excitation, a meniscus rises up along the inside and outside walls of the vertical electrodes to a height  $h_y$ , which is related to the wetting angle  $\theta_0$  by an equation attributed to Hagen.<sup>26</sup>

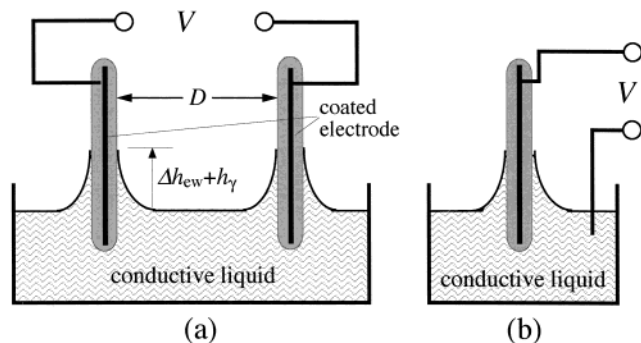
$$\sin \theta_0 \approx 1 + \frac{\rho_1 g h_y^2}{2\gamma_{vl}} \quad (21)$$

The accuracy of eq 21 requires that  $D \gg h_y$ . When voltage is applied, the liquid climbs further upward in response to electrowetting, that is, due to the reduction of the wetting angle in accordance with eq 9. No column of liquid spanning the electrodes rises up from the equilibrium level of the reservoir because the spacing is too large. In fact,

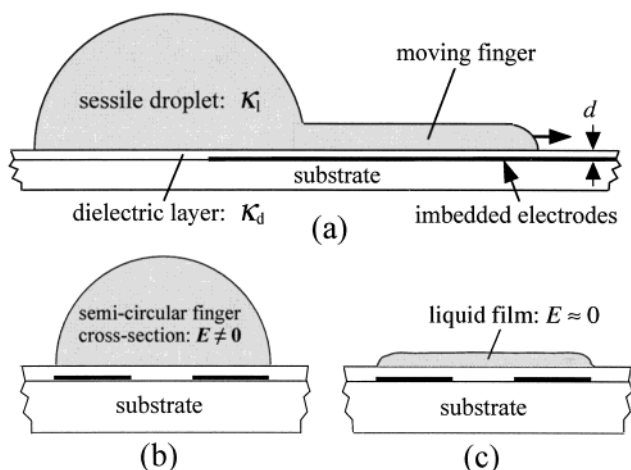
(24) Beni, G.; Hackwood, S.; Jackel, J. L. Continuous electrowetting effect. *Appl. Phys. Lett.* **1982**, *40*, 912–914 (see footnote 8).

(25) Jones, T. B.; Gunji, M.; Washizu, M. Dielectrophoretic liquid actuation and nanodroplet formation. *J. Appl. Phys.* **2001**, *89*, 1441–1448.

(26) Neumann, A. W. Über die Messmethodik zur Bestimmung grenzflächenenergetischer Grössen. *Z. Phys. Chem. Neune Folge* **1964**, *41*, 339–352.



**Figure 6.** Two unambiguous demonstrations of electrowetting phenomenon using coated electrodes and a conductive liquid. The voltage dependence of the contact angle fully accounts for  $\Delta h_{ew}$ . (a) Widely spaced electrodes ( $D \gg d$ ) with no significant, observable liquid height-of-rise. The liquid is electrically isolated. (b) Single coated electrode with water acting as one of the electrodes.



**Figure 7.** DEP microactuation with parallel electrode strips patterned on a substrate. (a) Side view with sessile liquid droplet placed at the left atop the surface electrodes. When sufficient voltage is applied, a finger emerges from the droplet and extends rapidly to the right toward the other end of the electrodes. (b) Cross section of the water finger for the condition  $f \gg f_c$ , showing the semicircular profile of water maintained by the DEP force. (c) Cross section of thin, slowly spreading liquid film observed at low frequencies ( $f \ll f_c$ ) where the electric field does not penetrate the liquid.

a similar rise in the liquid meniscus is observed if a single, coated electrode is immersed in water, with the water connected to the voltage source. See Figure 6b. For both of these simple setups, the electromechanical effect is limited to pure electrowetting, that is, the observable reduction of the contact angle and the associated change in the elevation of the contact line,  $\Delta h_{ew}$ . Parts a and b of Figure 6 are unambiguous representations of electrowetting, whereas the electromechanical height-of-rise depicted in Figures 2 and 4 is more properly thought of as an example of the net electromechanical force.

### DEP Actuation

DEP microactuation has one crucial distinction from electrowetting, namely, that the electric field lines penetrate the liquid and control its shape. This is why the behavior is strongly dependent on the frequency of the drive voltage. Figure 7 shows essential details of the DEP microactuation phenomenon. When an ac voltage is applied to the coplanar electrode strips embedded in the substrate, a finger of liquid emerges from the sessile

droplet and moves along the electrodes. See Figure 7a. Using deionized water ( $\sim 10^{-4}$  S/m) in air at some frequency  $f \geq \sim 60$  kHz, the finger has a roughly semicircular cross section, as shown in Figure 7b. On the other hand, for  $f \leq \sim 30$  kHz, the liquid spreads sluggishly in a thin film across the surface as shown in Figure 7c and no useable forward movement occurs. This critical frequency  $f_c \approx 60$  kHz, corresponding to the crossover from dominantly conductive ( $f < f_c$ ) to dielectric ( $f > f_c$ ) behavior of the water, can be predicted rather accurately using an RC circuit model<sup>25</sup> very similar to that used to obtain eq 11. Above the crossover, the electric field penetrates the liquid and overwhelms surface tension to create the semicircular profile of Figure 7b. This crossover frequency depends directly on  $d$ , the thickness of the dielectric layer. Experiments show that when this thickness is decreased from  $\sim 10$  to  $\sim 2$   $\mu\text{m}$ , the critical frequency decreases almost in direct proportion, and DEP actuation is observed as low as  $\sim 10$  kHz.

### Conclusion

It has been shown in this paper that a net force of electromechanical origin is exerted on conductive and insulative liquids by the electric field between parallel, planar electrodes which have been coated with a thin dielectric layer. This force, evaluated using either a lumped parameter (capacitive) model or the Maxwell stress tensor, is independent of contact angle and meniscus shape. It is operative irrespective of any field dependence of the contact angle. Appendix B shows that this electromechanical force, when expressed on a per unit area basis, is identical to what Prins, Welters, and Weekamp refer to as “electrocapillary pressure”, or ECP.<sup>22</sup> Because the net electromechanical force is not directly tied to the so-called electrowetting effect,<sup>16</sup> it seems advisable to restrict use of the term electrowetting to the observable reduction of the contact angle occurring in response to an electric field. This restriction establishes a clear and defensible connection of the term to recognized phenomenology. Not to do so inevitably confuses our understanding of the electrostatic forces at work in certain microfluidic actuation schemes now under investigation.<sup>9,21–23,25</sup> Electro-wetting may very well play an important role in microfluidics by helping to overcome stiction and contact angle hysteresis, but it is not responsible for the net force that moves liquid masses and droplets.

In this paper, we have used the electrowetting equation relating contact angle to the applied voltage, eqs 9 and 16, with Laplace’s capillarity equation to relate the net electromechanical force to the way the meniscus changes in response to the electrowetting effect. The consistency thus revealed is encouraging, though wholly expected. The apparent breakdown of the electrowetting equation as  $\theta_E \rightarrow 0$  is likely due to an overly restrictive interpretation of electrowetting as a phenomenon. In any case, it is clear that the electromechanical force model, used to predict the liquid height-of-rise, does not suffer this restriction.

**Acknowledgment.** The author gratefully acknowledges helpful discussion and criticisms offered by M. Washizu of Kyoto University and T. Blake and A. Clarke of Eastman Kodak in the United Kingdom. The Japan Society for the Promotion of Science, the U.S. National Science Foundation (Center for Global Partnership), the Center for Future Health (University of Rochester), and the National Institutes of Health supported this research.

### Appendix A

The Maxwell stress tensor provides a very general way to calculate the net electrical force using a surface

integral.<sup>27</sup> The z-directed electrical force is

$$f_z^e = \oint_{\Sigma} T_{zn}^e n_n dA \quad (A1)$$

Here,  $n_n$  is the unit normal to the surface,  $\Sigma$  is any closed surface enclosing all portions of the body harboring nonzero contributions to the body force, in our case, as represented by eq 2, and the Einstein summation convention has been employed.  $T_{zn}^e$  is the Maxwell stress tensor

$$T_{mn}^e = \epsilon E_{mn}^2 - \delta_{mm} \frac{1}{2} \epsilon E_k E_k \quad (A2)$$

where  $\delta_{mn}$  is the Kronecker delta function.

**Conductive Liquid.** For actuation of a conductive liquid as shown in Figure 3, the only nonzero contribution comes from the portion of the integral within the dielectric layer, where  $E_a = E_d = V_{\text{layer}}/d$ . Then, the surface integral of the stress tensor becomes

$$\oint_{\Sigma} T_{zn}^e n_n dA = -wd \left( -\frac{1}{2} \kappa_d \epsilon_0 E_d^2 \right) \quad (A3)$$

Dividing eq A3 by the electrode width  $w$  gives back eq 5, the expression for  $f_1^e$ , namely, the force per unit length of the contact line.

**Dielectric Liquid.** For a dielectric liquid, there are four contributions to the total force, as shown in Figure 5. The electric field expressions within these four regions are

$$E_a = \frac{V/2\kappa_d}{d/\kappa_d + D/2\kappa_1} \quad E_b = \frac{V/2\kappa_1}{d/\kappa_d + D/2\kappa_1} \quad (A4)$$

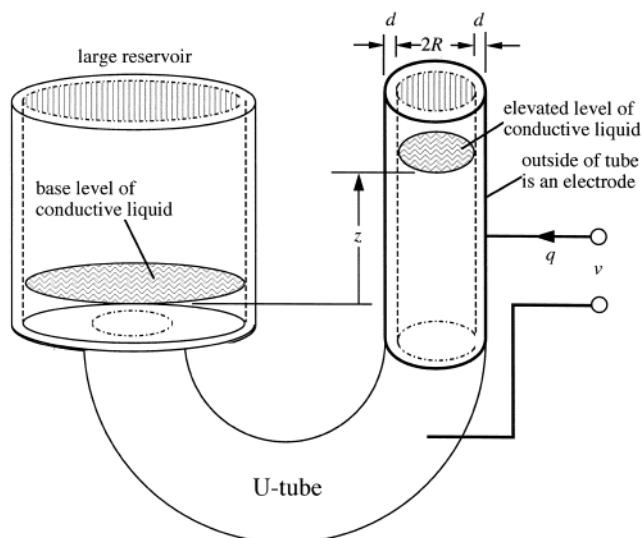
$$E_{a'} = \frac{V/2\kappa_d}{d/\kappa_d + D/2} \quad E_{b'} = \frac{V/2}{d/\kappa_d + D/2} \quad (A5)$$

Using these expressions, the stress integral may now be evaluated.

$$\oint_{\Sigma} T_{zn}^e n_n dA = dw \left( \frac{1}{2} \kappa_d \epsilon_0 E_a^2 \right) + \frac{Dw}{2} \left( \frac{1}{2} \kappa_1 \epsilon_0 E_b^2 \right) - dw \left( \frac{1}{2} \kappa_d \epsilon_0 E_{a'}^2 \right) - \frac{Dw}{2} \left( \frac{1}{2} \epsilon_0 E_{b'}^2 \right) \quad (A6)$$

Dividing this expression by the electrode width  $w$ , we reproduce eq 13 for the net force per unit length of the contact line.

(27) Woodson, H. H.; Melcher, J. R. *Electromechanical dynamics part II: Fields, Forces, and Motion*; Wiley: New York, 1968; Chapter 7.



**Figure 8.** Simple apparatus which demonstrates electrocapillary phenomena in a tube with a conductive liquid. The tube, of arbitrary cross section, has area  $A$  and perimeter  $l$ .

### Appendix B

Another example that can demonstrate both electrowetting and the electromechanical height-of-rise phenomenon is the case of a capillary with a single, coated electrode on the outside surface of the tube. Assume the tube is insulating with dielectric constant  $\kappa_d$  and thickness  $d$ . Refer to Figure 8. The conductive liquid acts as a moveable electrode. Interesting microfluidic applications based upon this simple scheme are under study.<sup>22</sup> Equation 5 for the force per unit length of contact line may be used to obtain an expression for the net force by setting  $V_{\text{layer}} = V$ . Letting the area and perimeter of the capillary cross section be  $A$  and  $l$ , respectively, then the force per unit area is

$$f_A^e = f_1^e l/A = \frac{\kappa_d \epsilon_0 V^2}{2d} \frac{l}{A} \quad (B1)$$

Prins et al. call this force the ECP, or electrocapillary pressure.<sup>22</sup> Their experimental approach is to measure a differential pressure, rather than the height-of-rise. Setting  $f_A^e$  equal to the gravitational head, we obtain

$$\Delta h = \frac{\kappa_d \epsilon_0 V^2}{2\rho_1 g d} \frac{l}{A} \quad (B2)$$

It is a simple exercise to verify the expression for  $f_1^e$ , the net force drawing the liquid upward, by a surface integration of the Maxwell stress tensor. Furthermore, the electrowetting equation, eq 9, also gives the same answer for the net force.