TWO-WAY RELAYING FOR ENERGY CONSTRAINED SYSTEMS: JOINT TRANSMIT POWER OPTIMIZATION

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Introduction

- Relaying systems mitigate effect of shadowing and extend coverage range.
- Two-way relaying compared with one-way relaying reduces loss in spectral efficiency.
- Two-way relaying system models:
  - Four phase two-way relaying
  - Three phase two-way relaying
  - Two phase two-way relaying
Previous work on joint transmit power optimization

Most relevant work:
(Zhang, et. al., ICC09) derived unconstrained optimal power allocation maximizing achievable rate and minimizing system outage probability.
(Chen, et. al., ICC09 & TWC10) found optimal power allocation among relay nodes such that an arbitrary weighted sum rate of all users is maximized.
(Agustin, et. al., ICC09) obtained optimal resource allocation such that sum rate is maximized, subject to a total power constraint for all terminals.

These works focus on improving sum-rate, subject to individual or total power constraint.

Our problem definition
We are interested in finding optimal power allocation that minimizes total transmit power, subject to end-to-end BER constraints at two terminals.
System model

- **Assumptions**
  - \( h_i \sim \mathcal{CN}(0, \sigma^{2}_{h_i}) \), \( \sigma^{2}_{h_i} = G/d_i^\varepsilon \), \( G = G_tG_r\lambda^2/(4\pi)^2 \), \( i \in (1, 2) \).
  - \( T_A \) knows \( h_1 \), \( T_B \) knows \( h_2 \), \( R \) knows both \( h_1, h_2 \).
  - \( P_1, P_2, P_r \) are average transmit power of \( T_A, T_B \) and \( R \).
  - \( n_A, n_B, n_r \sim \mathcal{CN}(0, N_0) \) are mutually independent.

- **Phase I**
  - \( T_A \) sends \( s_1 \in \{+1, -1\} \), \( T_B \) sends \( s_2 \in \{+1, -1\} \)
  - \( R \) receives \( r = h_1\sqrt{P_1}s_1 + h_2\sqrt{P_2}s_2 + n_r \)

- **Phase II**
  - \( R \) detects \( \hat{s}_1 \) and \( \hat{s}_2 \) based on \( r \).
  - \( R \) transmits \( x_r = \sqrt{P_r^B}\hat{s}_1 + \sqrt{P_r^A}\hat{s}_2 \)
    - \( P_r^A = \beta P_r \), \( P_r^B = (1 - \beta)P_r \), \( \beta \in [0, 1] \)
  - \( T_A \) and \( T_B \) receive \( y_A = h_1x_r + n_A \), \( y_B = h_2x_r + n_B \).
System model (con’d)

- Self-interference suppression at $T_A$:

$$z_A = y_A - h_1 \sqrt{P^B_r} s_1$$

$$= h_1 (\sqrt{P^B_r} (\hat{s}_1 - s_1) + \sqrt{P^A_r} \hat{s}_2) + n_A$$

When $\hat{s}_1 - s_1 \neq 0$ self-interference affects detection performance of $\hat{s}_2$ at $T_A$.

- Self-interference suppression at $T_B$:

$$z_B = y_B - h_2 \sqrt{P^A_r} s_2$$

$$= h_2 (\sqrt{P^B_r} \hat{s}_1 + \sqrt{P^A_r} (\hat{s}_2 - s_2)) + n_B$$

When $\hat{s}_2 - s_2 \neq 0$ self-interference affects detection performance of $\hat{s}_1$ at $T_B$. 
The end-to-end error probability $p_{b}^{T_A}$ is:

$$p_{b}^{T_A} = P(\hat{s}_2 = 1, s_2 = -1) + P(\hat{s}_2 = -1, s_2 = 1)$$

$$= P(\hat{s}_2 = 1, s_2 = -1 | \hat{s}_1 = s_1) P(\hat{s}_1 = s_1) \quad \text{without self-interference}$$

$$(a)$$

$$+ P(\hat{s}_2 = 1, s_2 = -1 | \hat{s}_1 \neq s_1) P(\hat{s}_1 \neq s_1) \quad \text{with self-interference}$$

$$(b)$$

$$+ P(\hat{s}_2 = -1, s_2 = 1 | \hat{s}_1 = s_1) P(\hat{s}_1 = s_1) \quad \text{without self-interference}$$

$$(a)$$

$$+ P(\hat{s}_2 = -1, s_2 = 1 | \hat{s}_1 \neq s_1) P(\hat{s}_1 \neq s_1) \quad \text{with self-interference}$$

$$(b)$$

where $\hat{s}_2$ is the estimate of $\hat{s}_2$ at $T_A$. 
We have:

- \( P(\hat{s}_1 \neq s_1) = 1 - P(\hat{s}_1 = s_1) = \delta_1 \)

- Term (a) is:

\[
P(\hat{s}_2 = 1, s_2 = -1 | \hat{s}_1 = s_1, \hat{s}_2 = 1) P(\hat{s}_2 = 1) + \\
P(\hat{s}_2 = 1, s_2 = -1 | \hat{s}_1 = s_1, \hat{s}_2 = -1) P(\hat{s}_2 = -1)
\]

\[= \frac{1}{2} [\delta_3 (1 - \delta_2) + \delta_2 (1 - \delta_3)]\]

- Term (b) is:

\[
P(\hat{s}_2 = 1, s_2 = -1 | \hat{s}_1 \neq s_1, \hat{s}_2 = 1) P(\hat{s}_2 = 1) + \\
P(\hat{s}_2 = 1, s_2 = -1 | \hat{s}_1 \neq s_1, \hat{s}_2 = -1) P(\hat{s}_2 = -1)
\]

\[= \frac{1}{2} [\delta_3 (1 - \delta_4) + \delta_5 (1 - \delta_3)]\]
The end-to-end error probability $p_{b}^{TA}$ is:

$$
p_{b}^{TA} = (1 - \delta_1)[(1 - \delta_2)\delta_3 + (1 - \delta_3)\delta_2] + \delta_1[(1 - \delta_4)\delta_3 + (1 - \delta_3)\delta_5]
$$

where $\delta_i = \frac{1}{2} \left( 1 - \sqrt{\frac{\bar{\gamma}_i}{1 + \bar{\gamma}_i}} \right)$ for $i = 1, \ldots, 5$.

$$
\bar{\gamma}_1 = \frac{P_1 d_1^{-\varepsilon}}{N_0}, \quad \bar{\gamma}_2 = \frac{P_r^A d_1^{-\varepsilon}}{N_0}, \quad \bar{\gamma}_3 = \frac{P_2 d_2^{-\varepsilon}}{N_0}
$$

$$
\bar{\gamma}_4 = \frac{(2 \sqrt{P_r^B} + \sqrt{P_r^A})^2 d_1^{-\varepsilon}}{N_0}, \quad \bar{\gamma}_5 = \frac{(2 \sqrt{P_r^B} - \sqrt{P_r^A})^2 d_1^{-\varepsilon}}{N_0}
$$
The optimization problem $\mathcal{OP}_1$

$\mathcal{OP}_1$: We minimize the total transmit power subject to end-to-end BER constraint

$$\begin{align*}
\text{min} & \quad \mathcal{P}_1 + \mathcal{P}_r + \mathcal{P}_2 \\
\text{s.t.} & \quad C_1: \quad \rho_b^A \leq \rho_b, \quad \rho_b^B \leq \rho_b \\
& \quad C_2: \quad \mathcal{P}_1, \mathcal{P}_2 > 0, \quad \mathcal{P}_r^A = \beta \mathcal{P}_r > 0, \quad \mathcal{P}_r^B = (1 - \beta) \mathcal{P}_r > 0
\end{align*}$$

- When designing an energy constrained network, we tend to maximize average node life time $T$:

  $$T = \frac{1}{3} \left( \frac{\mathcal{E}}{\mathcal{P}_1} + \frac{\mathcal{E}}{\mathcal{P}_2} + \frac{\mathcal{E}}{\mathcal{P}_r} \right) \geq \frac{\mathcal{E}}{\frac{1}{3}(\mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_r)}$$

  minimizing $\mathcal{P}_1 + \mathcal{P}_2 + \mathcal{P}_r$ maximizes lower bound on $T$.

- The closed-form solutions of $\mathcal{OP}_1$ are mathematically intractable. We numerically find the optimal solutions.
The optimization problem $\mathcal{OP}_2$

- At high SNR, the end-to-end BER constraints can be approximated as:

$$p_b^{T_A} \approx \frac{N_0}{4} P_1 d_1^{-\epsilon} + \frac{N_0}{4} P_r^B d_2^{-\epsilon}$$

$$p_b^{T_B} \approx \frac{N_0}{4} P_2 d_2^{-\epsilon} + \frac{N_0}{4} P_r^A d_1^{-\epsilon}$$

which are two concave functions.

$\mathcal{OP}_2$: The optimization problem $\mathcal{OP}_1$ can be reduced to $\mathcal{OP}_2$

<table>
<thead>
<tr>
<th>$T_A \rightarrow T_B$</th>
<th>$T_B \rightarrow T_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\min$ $P_1 + P_r^B$</td>
<td>$\min$ $P_2 + P_r^A$</td>
</tr>
<tr>
<td>$s.t.$ $C_1: p_b^{T_A} \leq p_b$</td>
<td>$s.t.$ $C_1: p_b^{T_B} \leq p_b$</td>
</tr>
<tr>
<td>$C_2: P_1 &gt; 0, P_r^B &gt; 0$</td>
<td>$C_2: P_2 &gt; 0, P_r^A &gt; 0$</td>
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Solving the optimization problem $\mathcal{OP}_2$

- The Lagrangian of the optimization problem for $\mathbb{T}_B \rightarrow \mathbb{T}_A$ is:

$$g(P_1, P_r^B) = P_1 + P_r^B - \lambda_1 \left( \frac{N_0}{4P_1d_1^{-\varepsilon}} + \frac{N_0}{4P_r^B d_2^{-\varepsilon}} - p_b \right)$$

Karush-Kuhn-Tucker (KKT) conditions

$$C_1 : \quad 1 - \lambda_1 \frac{N_0}{4P_1^2 d_1^{-\varepsilon}} = 0 \quad C_2 : \quad 1 - \lambda_1 \frac{N_0}{4P_r^{B^2} d_2^{-\varepsilon}} = 0$$

$$C_3 : \quad \lambda_1 \left( \frac{N_0}{4P_1 d_1^{-\varepsilon}} + \frac{N_0}{4P_r^B d_2^{-\varepsilon}} - p_b \right) = 0$$

$$C_4 : \quad \frac{N_0}{4P_1 d_1^{-\varepsilon}} + \frac{N_0}{4P_r^B d_2^{-\varepsilon}} \leq p_b$$
Solving the KKT conditions

Solving KKT conditions we have:

\[ P_1^* = \frac{N_0}{4\rho_b} (d_1^{\varepsilon} + \sqrt{d_1^{\varepsilon}d_2^{\varepsilon}}) \quad P_r^B = \frac{N_0}{4\rho_b} (d_2^{\varepsilon} + \sqrt{d_1^{\varepsilon}d_2^{\varepsilon}}) \]

\[ P_2^* = \frac{N_0}{4\rho_b} (d_2^{\varepsilon} + \sqrt{d_1^{\varepsilon}d_2^{\varepsilon}}) \quad P_r^A = \frac{N_0}{4\rho_b} (d_1^{\varepsilon} + \sqrt{d_1^{\varepsilon}d_2^{\varepsilon}}) \]

The optimal total transmit power is:

\[ P_{opt} = P_1^* + P_2^* + P_r^* = \frac{N_0}{2\rho_b} (d_1^{\varepsilon} + d_2^{\varepsilon} + 2\sqrt{d_1^{\varepsilon}d_2^{\varepsilon}}) \]

Since \( \frac{P_r^A}{\beta} = \frac{P_r^B}{1-\beta} \) we have \( \beta = \frac{\sqrt{d_1^{\varepsilon}}}{\sqrt{d_1^{\varepsilon} + \sqrt{d_2^{\varepsilon}}}} \), implying that \( \beta \) increases as \( \mathbb{R} \) moves away from \( T_A \) towards \( T_B \).
The optimization problem $\mathcal{OP}_3$

$\mathcal{OP}_3$: We minimize $\mathcal{P}'_i^*$ given per link BER constraint

$$
\begin{align*}
\min & \quad \mathcal{P}'_i \\
\text{s.t.} & \quad C_1 : \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_i}{1 + \bar{\gamma}_i}} \right) \leq p_b/2, \quad C_2 : \mathcal{P}'_i > 0
\end{align*}
$$

where $\mathcal{P}'_i \in \{ \mathcal{P}'_1, \mathcal{P}'_2, \mathcal{P}'_r^{A'}, \mathcal{P}'_r^{B'} \}$

- We find:
  $$
  \begin{align*}
  \mathcal{P}'_1 &= \frac{N_0}{2p_b d_1^{-\epsilon}}, \\
  \mathcal{P}'_2 &= \frac{N_0}{2p_b d_2^{-\epsilon}}, \\
  \mathcal{P}'_r^{A'} &= \frac{N_0}{2p_b d_2^{-\epsilon}}, \\
  \mathcal{P}'_r^{B'} &= \frac{N_0}{2p_b d_1^{-\epsilon}}
  \end{align*}
  $$

- The optimal total transmit power is:
  $$
  \mathcal{P}_{ind} = \mathcal{P}'_1 + \mathcal{P}'_2 + \mathcal{P}'_r = \frac{N_0}{p_b} (d_1^{\epsilon} + d_2^{\epsilon})
  $$

- Since $\frac{\mathcal{P}'_r^{A'}}{\beta} = \frac{\mathcal{P}'_r^{B'}}{1-\beta}$ we have $\beta = \frac{d_1^{\epsilon}}{d_1^{\epsilon} + d_2^{\epsilon}}$, implying that $\beta$ increases as $R$ moves away from $T_A$ towards $T_B$. 


Comparison of $\text{OP}_2$ and $\text{OP}_3$

- **Power reduction ratio:**

\[
\frac{P_{\text{opt}}}{P_{\text{ind}}} = \frac{1}{2} \left( \sqrt{d_1^\varepsilon} + \sqrt{d_2^\varepsilon} \right)^2 \frac{d_1^\varepsilon + d_2^\varepsilon}{2}
\]

- **Remarks:**
  - The ratio is independent of $N_0$;
  - The ratio becomes one when $d_1 = d_2$;
  - The ratio approaches $\frac{1}{2}$ when $d_1/d_2$ or $d_2/d_1$ is sufficiently high or path loss exponent $\varepsilon$ is large;
  - The approximation is accurate for low $p_b$ values.
Simulation setup

1. Linear network topology
2. \[ \frac{d_1}{d_1 + d_2} = \eta \]
3. \( G = 1, \sigma^2_{h_i} = 1/d_i^\varepsilon \)
4. Pathloss exponent \( \varepsilon = 2.5, 3.5 \)
5. \( N_0 = 3.98 \times 10^{-15} \text{ Watt} \)
6. Target BER \( p_b = 10^{-5} \)
Pathloss exponent $\varepsilon = 3.5$
Power reduction ratio $\frac{P_{opt}}{P_{ind}}$ vs. $\eta$

- Pathloss exponent $\varepsilon = 2.5, 3.5$
Power reduction ratio $P_{opt}/P_{ind}$ vs. $d_1$ and $d_2$

- Parameters: $\varepsilon = 3.5$
Considering self-interference, we derive end-to-end BER at $T_A$ and $T_B$.

We investigate the optimal power assignment among $T_A$, $R$, $T_B$ that minimizes the total transmit power, given end-to-end BER constraints at $T_A$ and $T_B$.

We show that total transmit power can be reduced up to 50% considering end-to-end BER constraint, as opposed to per link BER constraint.

The power saving is maximized when $R$ is relatively close to either of the two terminals, there is no power saving when $R$ is equal distant to $T_A$ and $T_B$. 