Topics:

• Sampling and quantization
• Analog to digital conversion
• Nyquist's theorem
• Band-limited reconstruction
• Aliasing
Sampling and Quantization

Sampling: Take “snapshots” of a waveform in time.
Quantization: Round off sample values to fixed levels.

$2^{16} = 65,536$ levels for CD audio

$R = 44,100$ for CD audio
Analog to digital conversion

- Track, sample and hold circuit
- Comparator

![Diagram of Analog to Digital Conversion]

- Sample n
- Sample n+1
- Sample n+2
- Comparison complete
- Value x(n)
- Value x(n+1)
- Value x(n+2)
- Hold
- Track
Track, sample and hold circuit

Capacitor voltage (and charge) follows input signal

Charge on capacitor (and thus the signal voltage) is held while comparator does its work.
Comparator

Comparator has a set of fixed levels against which the input voltage is compared.

$$V = \frac{Q}{C}$$

Binary output value: 1110

This example: 1101 → 13

4 bits → 16 levels

0000 → 0
0001 → 1
0010 → 2
0011 → 3
...
...
1111 → 15

N bits → $2^N$ levels

CD → 16 bits or 65,536 levels
Nyquist’s Theorem: To be able to accurately reconstruct a signal from its samples you need at least 2 samples per period for the highest frequency (sine wave) contained in the signal.

\[ R \geq 2 f_{\text{max}} \]

Critical sampling \( \rightarrow R = 2 f_{\text{max}} \)

Nyquist frequency = \( R/2 \) \( \rightarrow \) highest frequency that can be sampled.
Reconstruction from samples

But as far as we know the wave was a square wave!

Square wave “recipe”

\[ f_0, 3f_0, 5f_0, 7f_0 \]

High frequencies are removed during reconstruction.

Band-limited reconstruction – any frequencies above the Nyquist frequency are removed.

So after band-limited reconstruction we retrieve the original sine wave!
Aliasing occurs when a signal is sampled too slowly.\[ R < 2f_{\text{max}} \]
Aliasing continued

Example:
R = 8 samples per second

<table>
<thead>
<tr>
<th>Signal Frequency</th>
<th>Reconstructed Frequency</th>
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<tbody>
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Aliasing continued

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Audio ADC Sample Rate

• We need to capture 20 Hz → 20 kHz
• Sample rate 44,100 – standard CD quality
  – 48k, 96k, up to 192k for pro-audio
  – As low as 8k for voice

• Anti-aliasing filter
  – Make sure that there are no signals above one-half of the sample rate

![Graphs showing ideal and real response functions, with a dashed line indicating the move out cut-off to compensate for slow roll-off.](image)
Quantization

Use an N-bit binary number to map the range $2V$ onto $2^N$ levels.

Each region has a size: \[
\frac{2V}{2^N} \Rightarrow \frac{\text{Range}}{\# \text{levels}}
\]

Typical Audio: Line level $\approx 3.472 \text{ V peak to peak}$

16 bits $\Rightarrow 2^{16} = 65,536$ levels $\Rightarrow 3.472/2^{16} = 53 \mu\text{V steps}$
Quantization “round-off” error

Maximum round-off error (Quantization error)

\[ Q.E. = \frac{1}{2} \left( \frac{2V}{2^N} \right) \]

Quantization is just like adding noise to the signal.

\[
\frac{Signal}{Quantization \ Noise} = \frac{V}{\frac{1}{2} \left( \frac{2V}{2^N} \right)} = 2^N
\]
**SQNR** (Signal to quantization Noise Ratio)

SQNR in decibels (for N-bit quantization):

\[
SQNR(\text{dB}) = 20 \log_{10}(2^N) = 20N \log_{10}(2) \\
\log_{10}(2) = 0.3010 \\
SQNR(\text{dB}) \approx 6N
\]

Each additional bit gives a 6 dB increase in SQNR

16 bits $\rightarrow$ 96 dB

Refer back to the Fletcher-Munson curves
3-6 Equal-loudness contours of the human ear. These contours reveal the relative lack of sensitivity of the ear to bass tones, especially at lower sound levels. Inverting these curves give the frequency response of the ear in terms of loudness level. (After Robinson and Dadson.)