Introduction to Audio and Music Engineering
Lecture 17

Topics:

- Guitar pickups
- Guitar tone and volume controls
- Wah pedal – variable bandpass filter
- Amplifier project
- Tone control
- Gain control – distortion
- Speaker cabinet
5,000 → 6,000 turns of 44 AWG wire

44 AWG ≈ 0.05 mm (2x dia of human hair)
Electrical model of a guitar pickup

6000 turns
L \approx 2 \text{ Henry}
C \approx 120 \text{ pF}
R \approx 5000 \text{ } \Omega \text{ (DC resistance)*}

v_s = \text{voltage signal}
\text{induced in coil}
\text{from string motion}

* Effective resistance at audio frequencies \approx 10x \text{ DC resistance due to losses in the magnets}
**Guitar pickup analysis:** Find the output voltage, $v_{out}$

\[ i = \frac{v_s}{R + j\omega L + \frac{1}{j\omega C}} \]

\[ v_{out} = i \frac{1}{j\omega C} = v_s \frac{1}{R + j\omega L + \frac{1}{j\omega C}} \]

**Resonant frequency**

\[ \omega_r^2 = \frac{1}{LC} \]

**“Q” quality factor**

\[ Q = \frac{1}{\omega r RC} = \frac{1}{2\pi} \frac{T}{RC} \]

$T$ is the period of the resonance of the circuit

\[ v_{out} = v_s \frac{1}{1 - \omega^2/\omega_r^2 + j\omega/(\omega r Q)} \]

\[ |v_{out}| = v_s \frac{1}{\left[1 - \omega^2/\omega_r^2\right]^2 + \omega^2/(\omega_r^2 Q^2)^2}^{1/2} \]
Frequency response of pickups

![Frequency response graph](image)

- **7300 turns**
  - 3 Henrys
  - 1.80E-10 Farads
  - 60,000 Ohms

- **6000 turns**
  - 2 Henrys
  - 1.20E-10 Farads
  - 60,000 Ohms

- **5200 turns**
  - 1.5 Henrys
  - 9.00E-11 Farads
  - 60,000 Ohms

**Response** vs. **Frequency (kHz)**
Guitar tone & volume control circuit

4-way Selector switch
1) Bridge (lead) pickup
2) Neck (rhythm) pickup
3) Parallel combination
4) Series combination
Analysis of pickup and control circuit ...

Pickup model

\[ Z_p = \frac{(j \omega L_p + R_p) \cdot \frac{1}{j \omega C_p}}{j \omega L_p + R_p + \frac{1}{j \omega C_p}} = \frac{j \omega L_p + R_p}{1 - \omega^2 L_p C_p + j \omega R_p C_p} \]

Volume control

\[ Z_t = R_t + \frac{1}{j \omega C_t} \]

\[ \frac{v_1}{v_0} = \frac{v_0}{1 - \omega^2 L_p C_p + j \omega R_p C_p} \text{ ... to assure same open circuit voltage} \]

\[ v_{out} = v_1 \cdot g \frac{Z_t // R_O}{Z_p + Z_t // R_O} \]
Tone control frequency response ...

\[ v_{out} = v_1 \cdot g \frac{Z_t//R_O}{Z_p + Z_t//R_O} \]
Wah Pedal

https://www.youtube.com/watch?v=kMqGuF8VoRo

http://www.youtube.com/watch?v=HyfKQhTRwkU
Wah Pedal – Variable Bandpass Filter

Typical Values

- \( R_0 = 68 \, \text{k}\Omega \)
- \( R_1 = 1.5 \, \text{k}\Omega \)
- \( R_2 = 33 \, \text{k}\Omega \)
- \( L = 0.5 \, \text{H} \)
- \( C = 0.01 \, \mu\text{F} \rightarrow 1 \, \mu\text{F} \)

“shelving” band-pass filter

Graph showing frequency response with different capacitance values.
Op Amp Wah Pedal

\[ I = j\omega C \cdot V \]

Feedback changes the apparent size of C, thereby shifting the resonant frequency.
Full Schematic

Tone

Gain

Volume

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Coursera Guitar Amplifier Kit DAA CAB Rev. 1
Tone control circuit

\[ G = \left| \frac{Z_f}{Z_{\text{in}}} \right| = \frac{1}{R_{\text{in}}} \left| \frac{(1 - g)R_f + \frac{gR_f}{1 + jg\omega R_f C_f}}{1 - g - \frac{1}{j\omega C_f} + gR_f} \right| \]

\[ Z_f = (1 - g)R_f + \frac{\frac{1}{j\omega C_f} \cdot gR_f}{1 + jg\omega R_f C_f} \]

\[ 0 \leq g \leq 1 \]

\[ C_f \]

\[ gR_f \]

\[ (1-g)R_f \]
Frequency response versus tone pot setting

\[ |G| = \frac{1}{R_{in}} \left( (1 - g)R_f + \frac{gR_f}{1 + j\omega R_f C_f} \right) \]
Gain control - distortion

Diode

\[ I = I_0 \left( e^{V/V_T} - 1 \right) \]

\[ V_T \approx 0.026 \text{ V} \]

\[ V_{in} = R_{in} \left[ I_0 \left( e^{-V_{out}/V_T} - 1 \right) - \frac{V_{out}}{R_f} \right] \]

KCL: \( i_1 = i_2 + i_3 \)

\[ \frac{V_{in} - 0}{R_{in}} = \frac{0 - V_{out}}{R_f} + I_0 \left( e^{(0-V_{out})/V_T} - 1 \right) \]

Can’t solve analytically for \( V_{out} \) but we can use a graphical technique.
Graphical solution for $V_{\text{out}}$ vs. $V_{\text{in}}$

\[ V_{\text{in}} = R_{\text{in}} \left[ I_0 \left( \frac{-V_{\text{out}}}{V_T} - 1 \right) - \frac{V_{\text{out}}}{R_f} \right] \]

Find $V_{\text{in}}$ as a function of $V_{\text{out}}$ but then plot it as $V_{\text{out}}$ vs. $V_{\text{in}}$

But don't forget that this is an inverting amplifier configuration!

That's why we plotted the magnitude of the transfer function.
Making the transfer function symmetrical

D₁ conducts for \( V_{\text{in}} > 0 \),
(V\(_{\text{out}}\) will be negative)

D₂ conducts for \( V_{\text{in}} < 0 \),
(V\(_{\text{out}}\) will be positive)

Initial slope (Gain) is determined by \( R_f/R_{\text{in}} \)
Speaker Cabinet Design

- Guitar amplifiers normally have an open back so the cabinet has little acoustical effect.

- Bass cabinet design – to obtain better bass response from the amplifier the cabinet should be properly sized.
Thiele-Small parameters for project speaker

**SPEAKER SPECIFICATIONS**

<table>
<thead>
<tr>
<th>Model Number:</th>
<th>55-2970</th>
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<tbody>
<tr>
<td><strong>GENERAL SPECIFICATIONS</strong></td>
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<tr>
<td>Nominal Diameter:</td>
<td>6.5 inch</td>
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<tr>
<td>Rated Impedance:</td>
<td>8 ohms</td>
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<tr>
<td>Operating Bandwidth:</td>
<td>60-8kHz (–3dB)</td>
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<tr>
<td>Power Handling Capacity:</td>
<td>50 Watts</td>
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<td>Sensitivity (1W/M):</td>
<td>88 dB (+3dB)</td>
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<tr>
<td>Voice Coil Diameter:</td>
<td>1 inch</td>
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</table>

| **THIELE-SMALL PARAMETERS** | |
| Resonance Frequency | Fs: 40 Hz |
| DC Resistance | Re: 7.2 ohm |
| Mechanical Q Factor | Qms: 3.66 |
| Electrical Q Factor | Qes: 0.4796 |
| Total Q Factor | Qts: 0.424 |
| Equivalent Cas air load | Vas: 26.93 L |
| Surface Area of Cone | Sd: 0.0139 m2 |
| Efficiency Bandwidth Product | EBP: 83.4 |
| Voice Coil Over Hang | X-max: 3.5 mm |
| Voice Coil Inductance | Le: 0.3875mH |

**PHYSICAL INFORMATION**

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<tbody>
<tr>
<td>Basket:</td>
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<td>Cone Material:</td>
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<td>Surrounder:</td>
<td>Rubber</td>
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<td>Dust Cap:</td>
<td>PP</td>
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<tr>
<td>Damper:</td>
<td>Cloth</td>
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Date: 2010/6/28
Following Rob’s Week 5 Lecture 1 example ...

\( f_s = 40 \text{ Hz} \)
\( Q_{ts} = 0.424 \)
\( V_{AS} = 26.93 \text{ liters} \)

Choose \( Q_{cb} = 1 \) so \( Q_{cb}/Q_{ts} = 2.36 \)

\[
\frac{V_{as}}{V_{box}} = \left(\frac{Q_{cb}}{Q_{ts}}\right)^2 - 1 \quad \frac{V_{as}}{V_{box}} = (2.36)^2 - 1 = 4.57
\]

\[
V_{box} = \frac{V_{AS}}{4.57} = \frac{26.93}{4.57} = 5.89 \text{ liters} = 5890 \text{ cm}^3
\]

Using the ratio, \( 0.618 : 1 : 1.618 \)
...we find approximately
\( 11 \text{ cm} \times 18 \text{ cm} \times 29 \text{ cm} \)