Introduction to Audio and Music Engineering

Lecture 11

Topics:

• Charge, Coulomb’s Law
• Current
• Electric fields
• Voltage
• Resistors, Ohm’s Law
• Voltage Divider
Electric Charge

Coulomb’s Law (1783)

\[ F = k \frac{q_1 q_2}{r^2} \text{ Newtons} \]

\[ k = 8.988 \times 10^9 \text{ Nt-m}^2/\text{Coul}^2 \]

1 Newton is the force required to accelerate 1 kg by 1 m/sec^2

Newton’s 2\textsuperscript{nd} Law

\[ F = ma \]

Charge can be + or –

Like charges repel

Opposite charges attract

For:

\[ q_1 = q_2 = 1 \text{ Coul} \]
\[ r = 1 \text{ meter} \]
\[ F = 8.988 \times 10^9 \text{ Nt} \]

That’s a huge Force!

Enough to levitate about 5000, 200 ton locomotives!
Problem: Coulomb’s Law

When you are unpacking objects packed in styrofoam “peanuts” the peanuts usually stick to everything! How many electrons would there need to be on a styrofoam peanut of size $1 \text{ cm}^3$ to be picked up by your hand from a distance of 5 cm? Assume that the styrofoam peanut and your hand both have the same amount of charge on them, but with opposite signs. (the density of styrofoam is about 0.035 gram/cm³ and the Earth’s gravitational acceleration is $g = 9.8 \text{ m/sec}^2$.

Answer: $6.1 \times 10^{13}$ electrons
For our purposes it’s not important that charge is made up of discrete particles – we will treat charge like a fluid that is infinitesimally divisible.

- $q \rightarrow$ surplus of electrons
+ $q \rightarrow$ deficit of electrons

1 Ampere = 1 Coulomb/sec

+ current corresponds to positive charge moving in the direction indicated by arrow

Positive direction

+ $q$ Positive current
- $q$ Negative current

- $q$ Negative current
+ $q$ Positive current
Another look at Coulomb’s Law ...

Electric Field: $E$

$$F = q_1 \left( k \frac{q_2}{r^2} \right) = q_1 E_2$$

where

$$E_2 = k \frac{q_2}{r^2}$$

Electric field lines are directed radially from the charge $q_2$

$$F = q_1 E_2$$

$$F = q_2 \left( k \frac{q_1}{r^2} \right) = q_2 E_1$$
Electric Field between two charged plates

Electric field points from +Q to -Q

\[ F = qE \]

To move the charge against the electric field force requires that we do work.

\[ W = F \times d \]

Work “Energy” Force Distance

\[ W = mg \sin \theta \times d \]
Force, Work and Energy

\[ W = qE \times d \]

- positive

\[ \Rightarrow \text{so work is done by the electric field on the charge (energy is added)} \]

\[ W = -qE \times d \]

- negative

\[ \Rightarrow \text{so work must be done to move the charge} \]

Remember: opposite charges attract.
Electric Potential (Voltage)

The charge is moving in the direction of the E-field force.

→ The E field does work on the charge (adds energy).

→ So we say that the charge moves from a higher to a lower electric potential. (like rolling down a hill)

The charge is moving against the direction of the E-field force.

→ Work must be done to move the charge against the force (spend energy).

→ So we say that the charge moves from a lower to a higher electric potential.
Alessandro Volta

After whom the Volt is named ...

- Lived 1745 – 1827
- Born in Como, Italy
- Professor of experimental physics at University of Pavia for 40 years
- Invented the Voltaic Pile in 1800
Electric Potential (Voltage)

\[ \Delta V = V_2 - V_1 \]

Work done by the Electric field to bring a charge from potential \( V_2 \) to \( V_1 \)

\[ W = q\Delta V = q(V_2 - V_1) \]

So, if \( q \) is positive and \( V_2 > V_1 \) the field does work (adds energy) to the charge.
If $q$ is positive and $V_2 < V_1$ we have to do work to move the charge.

Only the difference in potential matters – we can set the zero of voltage any place we wish.

So rather than writing $\Delta V$ we just use $V$ for the voltage (potential) difference between two points.
Ohm’s Law

Resistor $R$

$+ \quad \quad -$

$I$

$V = IR \quad \text{or} \quad I = V/R$

For a fixed voltage: higher resistance $\rightarrow$ less current

Analogy: Fluid flowing through a pipe:

$p_2$ \hspace{1cm} large flow resistance \hspace{1cm} $p_1$

$p_2$ \hspace{1cm} small flow resistance \hspace{1cm} $p_1$

Small flow

Large flow
Georg Simon Ohm

• Lived 1789 – 1854
• Discovered Ohm’s law while teaching high school in Cologne
• Published in “The Galvanic Circuit Investigated Mathematically” in 1827
• His work was not appreciated until 1841 when he won the Copley Medal from the Royal Society
A first Simple Circuit

The battery
→ source of voltage
(potential difference)

A first simple circuit: how much current flows through R?

Voltage across R is V so by Ohm’s law

\[ I = \frac{V}{R} \]
Kirchhoff’s Voltage Law

Kirchhoff’s Voltage Law:
The sum of the voltages going around a closed path in a circuit is zero.

\[ \sum V_n = 0 \]
Gustav Kirchhoff

- Lived 1824 – 1887
- Born in Konigsburg, East Prussia
- Formulated his circuit laws in 1845 while he was still a student
- Collaborated with Bunsen to discover Rubidium and Caesium
- Made contributions to spectroscopy and optics
"Solving" the circuit with KVL

\[ \sum_{n} V_n = 0 \]

Start at the point \( a \) and proceed clockwise.
Voltage increases by \( V \) (go from \(-\) to \(+\))
Then voltage drops across resistor (+ to -)

\[ V - IR = 0 \quad V = IR \rightarrow I = V/R \]

This is a lot of formality just to retrieve Ohm's Law!
Voltage Divider

Find the current $I$
Find the voltage at $b$

\[
KVL \rightarrow V - IR_1 - IR_2 = 0
\]

\[
I = \frac{V}{R_1 + R_2}
\]

Series resistors: $R_{\text{series}} = R_1 + R_2$

Voltage divider: $V_b = IR_2 = V \frac{R_2}{R_1 + R_2}$

Special Cases:
$R_1 = R_2$ ; $V_b = \frac{1}{2} V$
$R_2 \gg R_1$ ; $V_b \rightarrow V$