CHAPTER 10

Section 10-1 to 10-3: Laplace Transform and Its Properties

Problem 10.1 Express each of the waveforms in Fig. P10.1 in terms of step functions and then determine its Laplace transfrom. [Recall that the ramp function is related to the step function by $r(t-T) = (t-T)\,u(t-T)$.] Assume that all waveforms are zero for $t < 0$.

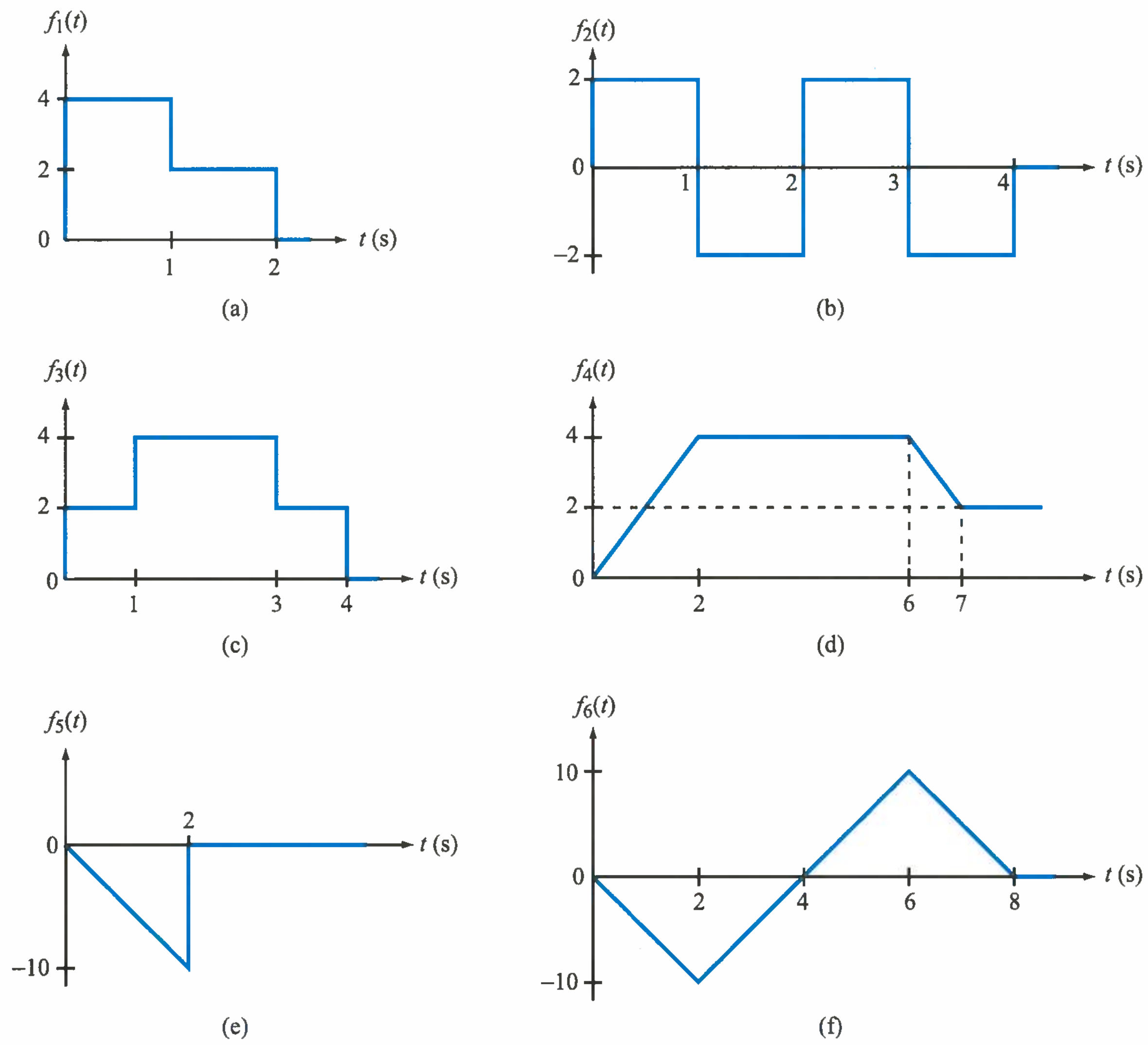

Figure P10.1: Waveforms for Problem 10.1.

Solution:

(a) $f_1(t) = 4u(t) - 2u(t-1) - 2u(t-2)$

$$\mathbf{F}_1(\mathbf{s}) = \frac{4}{\mathbf{s}} - \frac{2e^{-\mathbf{s}}}{\mathbf{s}} - \frac{2e^{-2\mathbf{s}}}{\mathbf{s}}.$$

(b) $f_2(t) = 2u(t) - 4u(t-1) + 4u(t-2) - 4u(t-3) + 2u(t-4)$

$$\mathbf{F}_2(\mathbf{s}) = \frac{2}{\mathbf{s}} - \frac{4e^{-\mathbf{s}}}{\mathbf{s}} + \frac{4e^{-2\mathbf{s}}}{\mathbf{s}} - \frac{4e^{-3\mathbf{s}}}{\mathbf{s}} + \frac{2e^{-4\mathbf{s}}}{\mathbf{s}}.$$

(c) $f_3(t) = 2u(t) + 2u(t-1) - 2u(t-3) - 2u(t-4)$

$$\mathbf{F}_3(\mathbf{s}) = \frac{2}{\mathbf{s}} + \frac{2e^{-\mathbf{s}}}{\mathbf{s}} - \frac{2e^{-3\mathbf{s}}}{\mathbf{s}} - \frac{2e^{-4\mathbf{s}}}{\mathbf{s}}.$$

(d) $f_4(t) = 2t\, u(t) - 2(t-2)\, u(t-2) - 2(t-6)\, u(t-6) + 2(t-7)\, u(t-7)$

$$\mathbf{F}_4(\mathbf{s}) = \frac{2}{\mathbf{s}^2} - \frac{2e^{-2\mathbf{s}}}{\mathbf{s}^2} - \frac{2e^{-6\mathbf{s}}}{\mathbf{s}^2} + \frac{2e^{-7\mathbf{s}}}{\mathbf{s}^2}.$$

(e) $f_5(t) = -5t\, u(t) + 10u(t-2) + 5(t-2)\, u(t-2)$

$$\mathbf{F}_5(\mathbf{s}) = \frac{-5}{\mathbf{s}^2} + \frac{10e^{-2\mathbf{s}}}{\mathbf{s}} + \frac{5e^{-2\mathbf{s}}}{\mathbf{s}^2}.$$

(f) $f_6(t) = -5t\, u(t) + 10(t-2)\, u(t-2) - 10(t-6)\, u(t-6) + 5(t-8)\, u(t-8)$

$$\mathbf{F}_6(\mathbf{s}) = \frac{-5}{\mathbf{s}^2} + \frac{10e^{-2\mathbf{s}}}{\mathbf{s}^2} - \frac{10e^{-6\mathbf{s}}}{\mathbf{s}^2} + \frac{5e^{-8\mathbf{s}}}{\mathbf{s}^2}.$$

Problem 10.3 Evaluate each of the following integrals:

(a) $G_1 = \int_{-\infty}^{\infty} (3t^3 + 2t^2 + 1)[\delta(t) + 4\delta(t-2)]\, dt$

(b) $G_2 = \int_{-2}^{4} 4(e^{-2t} + 1)[\delta(t) - 2\delta(t-2)]\, dt$

(c) $G_3 = \int_{-20}^{20} 3(t\cos 2\pi t - 1)[\delta(t) + \delta(t-10)]\, dt$

Solution:

(a)

$$\begin{aligned} G_1 &= \int_{-\infty}^{\infty} (3t^3 + 2t^2 + 1)[\delta(t) + 4\delta(t-2)]\, dt \\ &= (3t^3 + 2t^2 + 1)|_{t=0} + 4(3t^3 + 2t^2 + 1)|_{t=2} \\ &= 1 + 4(3 \times 8 + 2 \times 4 + 1) = 133. \end{aligned}$$

(b)

$$\begin{aligned} G_2 &= \int_{-2}^{4} 4(e^{-2t} + 1)[\delta(t) - 2\delta(t-2)]\, dt \\ &= 4(e^{-2t} + 1)|_{t=0} - 2 \times 4(e^{-2t} + 1)|_{t=2} \\ &= 8 - 8(e^{-4} + 1) = -0.147. \end{aligned}$$

(c)

$$\begin{aligned} G_3 &= \int_{-20}^{20} 3(t\cos 2\pi t - 1)[\delta(t) + \delta(t-10)]\, dt \\ &= 3(t\cos 2\pi t - 1)|_{t=0} + 3(t\cos 2\pi t - 1)|_{t=10} \\ &= -3 + 27 = 24. \end{aligned}$$

Problem 10.4 Determine the Laplace transform of each of the following functions, by applying the properties given in Tables 10-1 and 10-2:

(a) $f_1(t) = 4te^{-2t}\, u(t)$

(b) $f_2(t) = 10\cos(12t + 60^\circ)\, u(t)$

(c) $f_3(t) = 12e^{-3(t-4)}\, u(t-4)$

(d) $f_4(t) = 30(e^{-3t} + e^{3t})\, u(t)$

(e) $f_5(t) = 16e^{-2t}\cos 4t\, u(t)$

(f) $f_6(t) = 20te^{-2t}\sin 4t\, u(t)$

Solution:

(a)

$$f_1(t) = 4te^{-2t}\, u(t)$$

$$\mathbf{F}(\mathbf{s}) = 4\,\frac{1}{(\mathbf{s}+2)^2}. \qquad \text{[Entry \#6, Table 10-2]}$$

(b)

$$f_2(t) = 10\cos(12t + 60^\circ)\, u(t)$$

$$\mathbf{F}_2(\mathbf{s}) = 10\left[\frac{\mathbf{s}\cos 60^\circ - 12\sin 60^\circ}{\mathbf{s}^2 + 144}\right] = \frac{5\mathbf{s} - 10.392}{\mathbf{s}^2 + 144}.$$

[Entry #12, Table 10-2]

(c)

$$f_3(t) = 12e^{-3(t-4)}\, u(t-4)$$

$$\mathbf{F}_3(\mathbf{s}) = \frac{12^{-4\mathbf{s}}}{\mathbf{s}+3}. \qquad \text{[Entry \#3a, Table 10-2]}$$

(d)

$$f_4(t) = 30(e^{-3t} + e^{3t})\, u(t)$$

$$\mathbf{F}_4(\mathbf{s}) = 30\left(\frac{1}{\mathbf{s}+3} + \frac{1}{\mathbf{s}-3}\right) = \frac{60\mathbf{s}}{\mathbf{s}^2 - 9}. \qquad \text{[Entry \#3, Table 10-2]}$$

(e)

$$f_5(t) = 16e^{-2t}\cos 4t\, u(t)$$

$$\mathbf{F}_5(\mathbf{s}) = 16\,\frac{\mathbf{s}+2}{(\mathbf{s}+2)^2 + 16}. \qquad \text{[Entry \#14, Table 10-2]}$$

(f)

$$f_6(t) = 20te^{-2t}\sin 4t\, u(t)$$
$$= t\, f_a(t)$$

with

$$f_a(t) = 20e^{-2t}\sin 4t\, u(t)$$

Application of entry #13 of Table 10-2 gives

$$\mathbf{F}_a(\mathbf{s}) = 20\,\frac{4}{(\mathbf{s}+2)^2+16} = \frac{80}{(\mathbf{s}+2)^2+16}\,.$$

Application of property 9 in Table 10-1 gives:

$$\begin{aligned}\mathbf{F}_6(\mathbf{s}) &= -\frac{d}{d\mathbf{s}}\,\mathbf{F}_a(\mathbf{s}) \\ &= -\frac{d}{d\mathbf{s}}\left(\frac{80}{(\mathbf{s}+2)^2+16}\right) = \frac{160(\mathbf{s}+2)}{[(\mathbf{s}+2)^2+16]^2}\,.\end{aligned}$$

Problem 10.5 Determine the Laplace transform of each of the following functions, by applying the properties given in Tables 10-1 and 10-2:

(a) $h_1(t) = 12te^{-3(t-4)}\, u(t-4)$

(b) $h_2(t) = 27t^2 \sin(6t - 60^\circ)\, u(t)$

(c) $h_3(t) = 10t^3 e^{-2t}\, u(t)$

(d) $h_4(t) = 5(t-6)\, u(t-3)$

(e) $h_5(t) = 10e^{-3t}\, u(t-4)$

(f) $h_6(t) = 4e^{-2(t-3)}\, u(t-4)$

Solution:

(a)

$$h_1(t) = 12te^{-3(t-4)}\, u(t-4) = th_a(t)$$

with

$$h_a(t) = 12e^{-3(t-4)}\, u(t-4),$$

whose transform is

$$\mathbf{H}_a(\mathbf{s}) = \frac{12e^{-4\mathbf{s}}}{\mathbf{s}+3} \qquad \text{[Entry \#3a, Table 10-2]}$$

Application of Property 9 in Table 10-1 leads to

$$\mathbf{H}_1(\mathbf{s}) = -\frac{d}{d\mathbf{s}}\mathbf{H}_a(\mathbf{s}) = -\frac{d}{d\mathbf{s}}\left(\frac{12e^{-4\mathbf{s}}}{\mathbf{s}+3}\right) = \left(\frac{48}{\mathbf{s}+3} + \frac{12}{(\mathbf{s}+3)^2}\right)e^{-4\mathbf{s}}.$$

(b)

$$h_2(t) = 27t^2 \sin(6t - 60^\circ)\, u(t) = t^2\, h_b(t)$$

with

$$h_b(t) = 27\sin(6t - 60^\circ)\, u(t),$$

whose transform is

$$\mathbf{H}_b(\mathbf{s}) = 27\frac{(-\mathbf{s}\sin 60^\circ + 6\cos 60^\circ)}{\mathbf{s}^2+36} = \frac{-23.38\mathbf{s}+81}{\mathbf{s}^2+36}.$$

Multiplication by t in the time domain corresponds to negative differentiation in the s-domain (property 9 of Table 10-1). Hence,

$$\mathbf{H}_2(\mathbf{s}) = \frac{d^2}{d\mathbf{s}^2}\mathbf{H}_b(\mathbf{s}) = \frac{d^2}{d\mathbf{s}^2}\left(\frac{23.38\mathbf{s}+81}{\mathbf{s}^2+36}\right) = \frac{46.76\mathbf{s}^3 + 486\mathbf{s}^2 - 5051\mathbf{s} - 5832}{(\mathbf{s}^2+36)^3}.$$

(c)

$$h_3(t) = 10t^3 e^{-2t}\, u(t) \\ = t^3\, h_c(t)$$

with

$$h_c(t) = 10e^{-2t}\, u(t),$$

whose transform is

$$\mathbf{H}_c(\mathbf{s}) = 10\,\frac{1}{\mathbf{s}+2} \qquad \text{[Entry \#3 in Table 10-2].}$$

Multiplication by t in the time domain is equivalent to negative differentiation in the s-domain (property 9 of Table 10-1). Hence,

$$\mathbf{H}_3(\mathbf{s}) = -\frac{d^3}{d\mathbf{s}^3}\, H_c(\mathbf{s}) \\ = -\frac{d^3}{d\mathbf{s}^3}\left(\frac{10}{\mathbf{s}+2}\right) = \frac{60}{(\mathbf{s}+2)^4}\,.$$

(d)

$$h_y = 5(t-6)\, u(t-3) \\ = 5t\, u(t-3) - 30u(t-3)$$

$$\mathbf{H}_4(\mathbf{s}) = -5\,\frac{d}{d\mathbf{s}}\left(\frac{e^{-3\mathbf{s}}}{\mathbf{s}}\right) - 30\,\frac{e^{-3\mathbf{s}}}{\mathbf{s}} \\ = \frac{15e^{-3\mathbf{s}}}{\mathbf{s}} + \frac{5e^{-3\mathbf{s}}}{\mathbf{s}^2} - 30\,\frac{e^{-3\mathbf{s}}}{\mathbf{s}} = \left(\frac{5}{\mathbf{s}^2} - \frac{15}{\mathbf{s}}\right) e^{-3\mathbf{s}}.$$

(e)

$$h_5(t) = 10e^{-3t}\, u(t-4) \\ = 10e^{-3(t-4)} \cdot e^{-12}\, u(t-4) \\ = 6.1 \times 10^{-5} e^{-3(t-4)}\, u(t-4)$$

Hence,

$$\mathbf{H}_5(\mathbf{s}) = \frac{6.1 \times 10^{-5} e^{-4\mathbf{s}}}{\mathbf{s}+3}\,.$$

(f)

$$h_6(t) = 4e^{-2(t-3)}\, u(t-4) \\ = 4e^{-2(t-4)} \cdot e^{-2} \cdot u(t-4) = 0.54e^{-2(t-4)}\, u(t-4)$$

$$\mathbf{H}_6(\mathbf{s}) = 0.54\,\frac{e^{-4\mathbf{s}}}{\mathbf{s}+2}\,.$$

Problem 10.9 Determine $f(0^+)$ and $f(\infty)$, given that

$$\mathbf{F}(\mathbf{s}) = \frac{12e^{-2\mathbf{s}}}{\mathbf{s}(\mathbf{s}+2)(\mathbf{s}+3)}\,.$$

Solution:

$$f(0^+) = \lim_{\mathbf{s}\to\infty} \mathbf{s}\,\mathbf{F}(\mathbf{s}) = \lim_{\mathbf{s}\to\infty} \left[\frac{12e^{-2\mathbf{s}}}{(\mathbf{s}+2)(\mathbf{s}+3)}\right] = 0.$$

$$f(\infty) = \lim_{\mathbf{s}\to 0} \mathbf{s}\,\mathbf{F}(\mathbf{s}) = \lim_{\mathbf{s}\to 0} \left[\frac{12e^{-2\mathbf{s}}}{(\mathbf{s}+2)(\mathbf{s}+3)}\right] = \frac{12}{6} = 2.$$

Section 10-5: Partial Fraction Expansion

Problem 10.11 Obtain the inverse Laplace transform of each of the following functions, by first applying the partial-fraction-expansion method:

(a) $\mathbf{F}_1(\mathbf{s}) = \dfrac{6}{(\mathbf{s}+2)(\mathbf{s}+4)}$

(b) $\mathbf{F}_2(\mathbf{s}) = \dfrac{4}{(\mathbf{s}+1)(\mathbf{s}+2)^2}$

(c) $\mathbf{F}_3(\mathbf{s}) = \dfrac{3\mathbf{s}^3+36\mathbf{s}^2+131\mathbf{s}+144}{\mathbf{s}(\mathbf{s}+4)(\mathbf{s}^2+6\mathbf{s}+9)}$

(d) $\mathbf{F}_4(\mathbf{s}) = \dfrac{2\mathbf{s}^2+4\mathbf{s}-16}{(\mathbf{s}+6)(\mathbf{s}+2)^2}$

Solution:

(a)

$$\mathbf{F}_1(\mathbf{s}) = \frac{6}{(\mathbf{s}+2)(\mathbf{s}+4)} = \frac{A_1}{\mathbf{s}+2} + \frac{A_2}{\mathbf{s}+4},$$

with

$$A_1 = (\mathbf{s}+2)\,\mathbf{F}_1(\mathbf{s})\big|_{\mathbf{s}=-2} = \left.\frac{6}{\mathbf{s}+4}\right|_{\mathbf{s}=-2} = \frac{6}{-2+4} = 3,$$

$$A_2 = (\mathbf{s}+4)\,\mathbf{F}_1(\mathbf{s})\big|_{\mathbf{s}=-4} = \left.\frac{6}{\mathbf{s}+2}\right|_{\mathbf{s}=-4} = \frac{6}{-4+2} = -3.$$

Hence,

$$\mathbf{F}_1(\mathbf{s}) = \frac{3}{\mathbf{s}+2} - \frac{3}{\mathbf{s}+4},$$

and

$$f_1(t) = 3(e^{-2t} - e^{-4t})\,u(t).$$

(b)

$$\mathbf{F}_2(\mathbf{s}) = \frac{4}{(\mathbf{s}+1)(\mathbf{s}+2)^2} = \frac{A}{\mathbf{s}+1} + \frac{B_2}{(\mathbf{s}+2)^2} + \frac{B_1}{(\mathbf{s}+2)},$$

with

$$A = (\mathbf{s}+1)\;\mathbf{F}_2(\mathbf{s})\big|_{\mathbf{s}=-1} = \left.\frac{4}{(\mathbf{s}+2)^2}\right|_{\mathbf{s}=-1} = 4,$$

$$B_2 = (\mathbf{s}+2)^2\;\mathbf{F}_2(\mathbf{s})\big|_{\mathbf{s}=-2} = \left.\frac{4}{\mathbf{s}+1}\right|_{\mathbf{s}=-2} = -4,$$

$$B_1 = \left.\frac{d}{ds}\left(\frac{4}{\mathbf{s}+1}\right)\right|_{\mathbf{s}=-2} = \left.\frac{-4}{(\mathbf{s}+1)^2}\right|_{\mathbf{s}=-2} = -4.$$

Hence,

$$\mathbf{F}_2(\mathbf{s}) = \frac{4}{\mathbf{s}+1} - \frac{4}{(\mathbf{s}+2)^2} - \frac{4}{\mathbf{s}+2},$$

and

$$\begin{aligned} f_2(t) &= [4e^{-t} - 4te^{-2t} - 4e^{-2t}]\, u(t) \\ &= 4[e^{-t} - (1+t)e^{-2t}]\, u(t). \end{aligned}$$

(c)

$$\begin{aligned} \mathbf{F}_3(\mathbf{s}) &= \frac{3\mathbf{s}^3 + 36\mathbf{s}^2 + 131\mathbf{s} + 144}{\mathbf{s}(\mathbf{s}+4)(\mathbf{s}^2+6\mathbf{s}+9)} \\ &= \frac{3\mathbf{s}^3 + 36\mathbf{s}^2 + 131\mathbf{s} + 144}{\mathbf{s}(\mathbf{s}+4)(\mathbf{s}+3)^2} \\ &= \frac{A_1}{\mathbf{s}} + \frac{A_2}{\mathbf{s}+4} + \frac{B_2}{(\mathbf{s}+3)^2} + \frac{B_1}{\mathbf{s}+3}, \end{aligned}$$

with

$$\begin{aligned} A_1 &= \mathbf{s}\ \mathbf{F}_3(\mathbf{s})\big|_{\mathbf{s}=0} = \left.\frac{3\mathbf{s}^3 + 36\mathbf{s}^2 + 131\mathbf{s} + 144}{(\mathbf{s}+4)(\mathbf{s}+3)^2}\right|_{\mathbf{s}=0} = 4, \\ A_2 &= (\mathbf{s}+4)\ \mathbf{F}(\mathbf{s})\big|_{\mathbf{s}=-4} = \left.\frac{3\mathbf{s}^3 + 36\mathbf{s}^2 + 131\mathbf{s} + 144}{\mathbf{s}(\mathbf{s}+3)^2}\right|_{\mathbf{s}=-4} = -1, \\ B_2 &= (\mathbf{s}+3)^2\ \mathbf{F}(\mathbf{s})\big|_{\mathbf{s}=-3} = \left.\frac{3\mathbf{s}^3 + 36\mathbf{s}^2 + 131\mathbf{s} + 144}{\mathbf{s}(\mathbf{s}+4)}\right|_{\mathbf{s}=-3} = 2, \\ B_1 &= \frac{d}{d\mathbf{s}}\left[\frac{3\mathbf{s}^3 + 36\mathbf{s}^2 + 131\mathbf{s} + 144}{\mathbf{s}(\mathbf{s}+4)}\right]\Bigg|_{\mathbf{s}=-3} \\ &= \left[\frac{9\mathbf{s}^2 + 72\mathbf{s} + 131}{\mathbf{s}(\mathbf{s}+4)} - \frac{(2\mathbf{s}+4)(3\mathbf{s}^3 + 36\mathbf{s}^2 + 131\mathbf{s} + 144)}{\mathbf{s}^2(\mathbf{s}+4)^2}\right]\Bigg|_{\mathbf{s}=-3} = 0. \end{aligned}$$

Hence,

$$\mathbf{F}_3(\mathbf{s}) = \frac{4}{\mathbf{s}} - \frac{1}{\mathbf{s}+4} + \frac{2}{(\mathbf{s}+3)^2},$$

and

$$f_3(t) = [4 - e^{-4t} + 2te^{-3t}]\, u(t).$$

(d)

$$\begin{aligned} \mathbf{F}_4(\mathbf{s}) &= \frac{2\mathbf{s}^2 + 4\mathbf{s} - 16}{(\mathbf{s}+6)(\mathbf{s}+2)^2} \\ &= \frac{A}{\mathbf{s}+6} + \frac{B_2}{(\mathbf{s}+2)^2} + \frac{B_1}{\mathbf{s}+2}, \end{aligned}$$

with

$$A = (\mathbf{s}+6)\ \mathbf{F}_4(\mathbf{s})\big|_{\mathbf{s}=-6} = \left.\frac{2\mathbf{s}^2 + 4\mathbf{s} - 16}{(\mathbf{s}+2)^2}\right|_{\mathbf{s}=-6} = 2,$$

$$B_2 = (\mathbf{s}+2)^2 \ \mathbf{F}_4(\mathbf{s})\big|_{\mathbf{s}=-2} = \left.\frac{2\mathbf{s}^2+4\mathbf{s}-16}{\mathbf{s}+6}\right|_{\mathbf{s}=-2} = -4,$$

$$B_1 = \frac{d}{d\mathbf{s}}\left[\frac{2\mathbf{s}^2+4\mathbf{s}-16}{\mathbf{s}+6}\right]\Bigg|_{\mathbf{s}=-2}$$

$$= \left.\frac{4\mathbf{s}+4}{\mathbf{s}+6} - \frac{(2\mathbf{s}^2+4\mathbf{s}-16)}{(\mathbf{s}+6)^2}\right|_{\mathbf{s}=-2} = 0.$$

Hence,

$$\mathbf{F}_4(\mathbf{s}) = \frac{2}{\mathbf{s}+6} - \frac{4}{(\mathbf{s}+2)^2},$$

and

$$f_4(t) = (2e^{-6t} - 4te^{-2t})\ u(t).$$

Sections 10-6 and 10-7: s-domain Analysis

Problem 10.15 Determine $v(t)$ in the circuit of Fig. P10.15, given that $v_s(t) = 2u(t)$ V, $R_1 = 1\ \Omega$, $R_2 = 3\ \Omega$, $C = 0.3689$ F, and $L = 0.2259$ H.

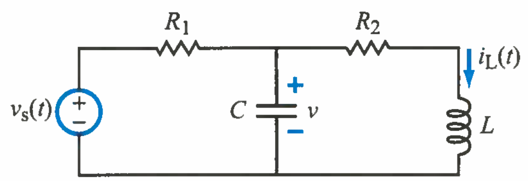

Figure P10.15: Circuit for Problem 10.15 and 10.16.

Solution: Transformation to the s-domain entails

$$v_s(t) = 20\,u(t) \quad \rightarrow \quad \mathbf{V}_s = \frac{2}{\mathbf{s}} \quad \text{(V)},$$

$$L \quad \rightarrow \quad \mathbf{s}L,$$

$$C \quad \rightarrow \quad \frac{1}{\mathbf{s}C}.$$

The s-domain circuit is shown in Fig. P10.15(a).

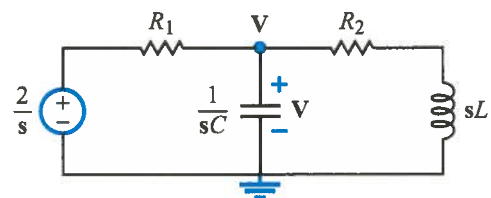

Figure P10.15(a)

At node **V**:

$$\frac{\mathbf{V} - \mathbf{V}_s}{R_1} + \frac{\mathbf{V}}{1/\mathbf{s}C} + \frac{\mathbf{V}}{R_2 + \mathbf{s}L} = 0,$$

which leads to

$$\mathbf{V} = \frac{2(R_2 + \mathbf{s}L)}{\mathbf{s}[R_1 LC\mathbf{s}^2 + (L + R_1R_2C)\mathbf{s} + (R_1 + R_2)]}$$
$$= \frac{24(3 + 0.2259\mathbf{s})}{\mathbf{s}(\mathbf{s}^2 + 16\mathbf{s} + 48)}$$
$$= \frac{72(1 + 0.0753\mathbf{s})}{\mathbf{s}(\mathbf{s}+4)(\mathbf{s}+12)}.$$

Partial fraction expansion gives

$$\mathbf{V} = \frac{A_1}{\mathbf{s}} + \frac{A_2}{\mathbf{s}+4} + \frac{A_3}{\mathbf{s}+12},$$

with

$$A_1 = \mathbf{s}\mathbf{V}\big|_{\mathbf{s}=0} = \left.\frac{72(1 + 0.0753\mathbf{s})}{(\mathbf{s}+4)(\mathbf{s}+12)}\right|_{\mathbf{s}=0} = 1.5,$$

$$A_2 = (\mathbf{s}+4)\mathbf{V}\big|_{\mathbf{s}=-4} = \left.\frac{72(1+0.0753\mathbf{s})}{\mathbf{s}(\mathbf{s}+12)}\right|_{\mathbf{s}=-4} = -1.56,$$

$$A_3 = (\mathbf{s}+12)\mathbf{V}\big|_{\mathbf{s}=-12} = \left.\frac{72(1+0.0753\mathbf{s})}{\mathbf{s}(\mathbf{s}+4)}\right|_{\mathbf{s}=-12} = 0.072.$$

Hence,

$$\mathbf{V} = \frac{1.5}{\mathbf{s}} - \frac{1.56}{\mathbf{s}+4} + \frac{0.072}{\mathbf{s}+12},$$

and the corresponding time-domain solution is

$$v(t) = [1.5 - 1.56e^{-4t} + 0.072e^{-12t}]\,u(t) \qquad \text{(V)}.$$