Problem 9.36 Design an active lowpass filter with a gain of 4, a corner frequency of 1 kHz, and a gain roll-off rate of −60 dB/decade.

Solution: The roll-off rate of −60 dB requires a three-stage LP filter, similar in design to that in Fig. 9-26. To secure positive gain, we need an additional fourth stage. Arbitrarily, we choose all resistors of the first three stages to be 10-kΩ resistors, and we realize the overall gain through the last stage.

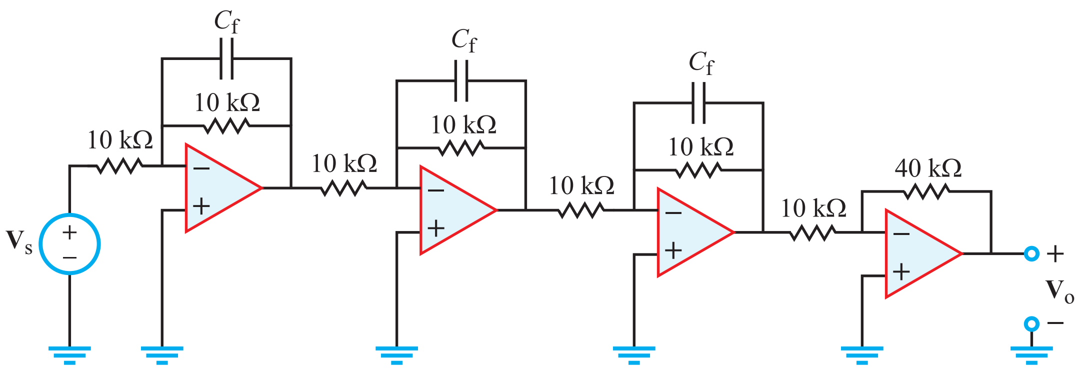


Figure P9.36(a)

The transfer function is given by

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = 4\left(\frac{1}{1 + j\omega/\omega_{c_1}}\right)^3,$$

with

$$\omega_{c_1} = \frac{1}{R_f C_f}.$$

The problem states that the corner frequency of the overall filter, which we will call ω_{c_3}, should be

$$\omega_{c_3} = 2\pi f_c = 2\pi \times 10^3 \text{ rad/s}.$$

According to Exercise 9.14,

$$\omega_{c_3} = 0.51\omega_{c_1}.$$

Hence,

$$\omega_{c_1} = \frac{\omega_{c_3}}{0.51} = \frac{2\pi \times 10^3}{0.51} = 12.32 \text{ krad/s},$$

and since $R_f = 10$ kΩ,

$$C_f = \frac{1}{R_f \omega_{c_1}} = 8.12 \text{ nF}.$$

The spectral response of the magnitude of $\mathbf{H}(\omega)$ is shown in Fig. P9.36(b).

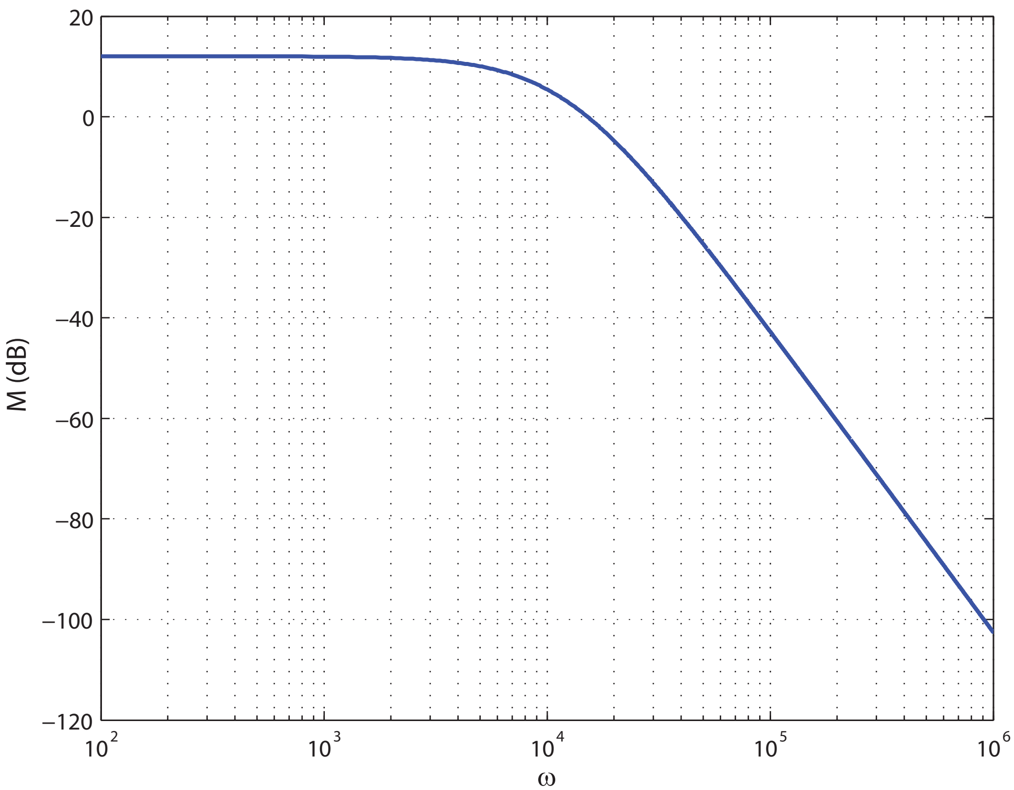
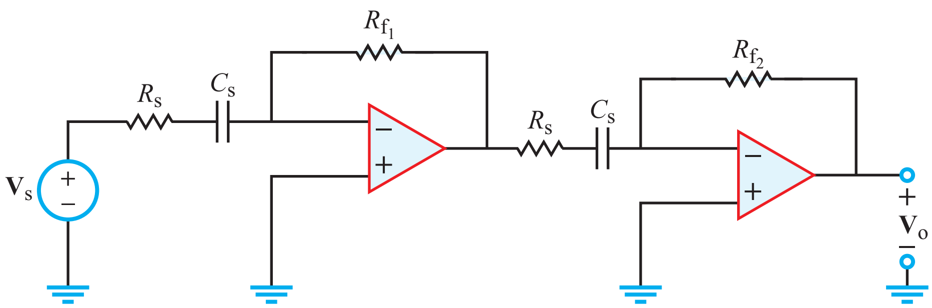


Figure P9.36(b)

Problem 9.37 Design an active highpass filter with a gain of 10, a corner frequency of 2 kHz, and a gain roll-off rate of 40 dB/decade.

Solution: To secure a roll-off rate of 40 dB/decade we need to use two stages of the circuit in Fig. 9-24.



The two stages have the same input impedances (R_s and C_s). We choose

$$R_{f_1} = R_s = 10\text{ k}\Omega, \qquad R_{f_2} = 100\text{ k}\Omega.$$

Consequently,

$$G_1 = -\frac{R_{f_1}}{R_s} = -1, \qquad G_2 = -\frac{R_{f_2}}{R_s} = -10.$$

The overall response is:

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_o}{\mathbf{V}_s} = G_1 G_2 \left(\frac{j\omega/\omega_{HP}}{1 + j\omega/\omega_{HP}} \right)^2 = 10 \left(\frac{j\omega/\omega_{HP}}{1 + j\omega/\omega_{HP}} \right)^2,$$

with

$$\omega_{HP} = \frac{1}{R_s C_s}.$$

The problem statement specifies a corner frequency $f_c = 2$ kHz with a corresponding angular frequency ω_c given by

$$\omega_c = 2\pi f_c = 4\pi \times 10^3 \text{ rad/s}.$$

By definition, ω_c is the angular frequency at which the magnitude of $\mathbf{H}(\omega)$ is equal to 0.707 of its maximum value. Thus, at $\omega = \omega_c$,

$$|\mathbf{H}(\omega_c)| = 10 \left| \left(\frac{j\omega_c/\omega_{HP}}{1 + j\omega_c/\omega_{HP}} \right)^2 \right| = 7.07,$$

which leads to

$$\frac{x^2}{1+x^2} = 0.707$$

with $x = \omega_c/\omega_{HP}$.

Solution of the above equation gives

$$x = 1.55.$$

Hence

$$\omega_{HP} = \frac{1}{R_s C_s} = 1.55\omega_c = 1.55 \times 4\pi \times 10^3 = 1.95 \times 10^4 \text{ rad/s},$$

and

$$C_s = \frac{1}{R_s \omega_{HP}} = \frac{1}{10^4 \times 1.95 \times 10^4} = 5.1 \times 10^{-9} \text{ F} = 5.1 \text{ nF}.$$

A plot of M [dB] is shown in Fig. P9.37(b).

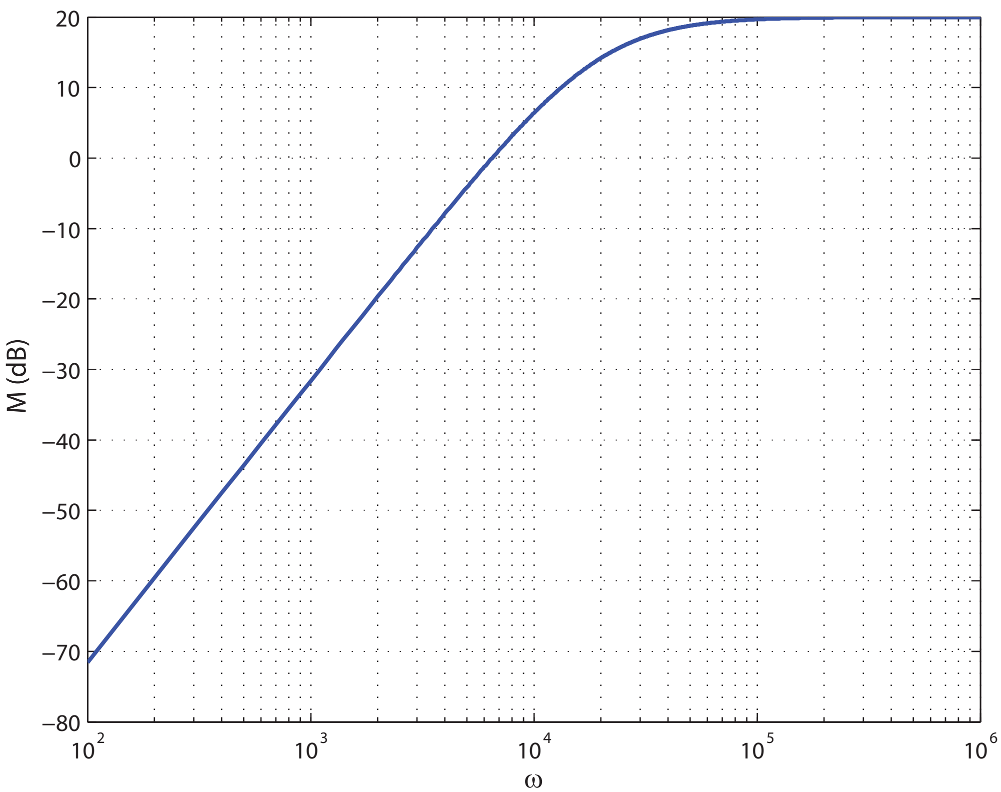


Figure P9.37(b)

Problem 9.38 The element values in the circuit of the second-order bandpass filter shown in Fig. P9.38 are: $R_{f_1} = 100$ kΩ, $R_{s_1} = 10$ kΩ, $R_{f_2} = 100$ kΩ, $R_{s_2} = 10$ kΩ, $C_{f_1} = 3.98 \times 10^{-11}$ F, $C_{s_2} = 7.96 \times 10^{-11}$ F. Generate a spectral plot for the magnitude of $\mathbf{H}(\omega) = \mathbf{V}_o/\mathbf{V}_s$. Determine the frequency locations of the maximum value of M [dB] and its half-power points.

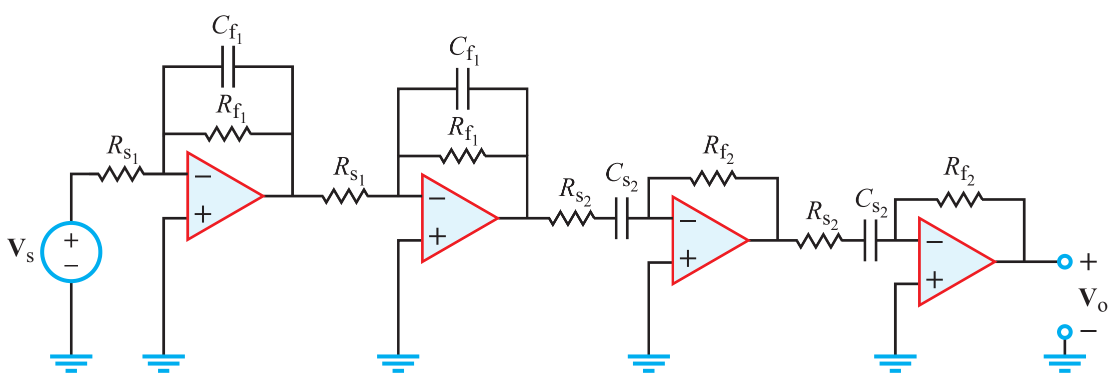


Figure P9.38: Circuit for Problem 9.38.

Solution: The overall transfer function is given by

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_{out}}{\mathbf{V}_s} = G_{LP}^2 G_{HP}^2 \left(\frac{1}{1 + j\omega/\omega_{LP}}\right)^2 \left(\frac{j\omega/\omega_{HP}}{1 + j\omega/\omega_{HP}}\right)^2, \qquad (1)$$

with

$$G_{LP} = -\frac{R_{f_1}}{R_{s_1}} = -10,$$

$$G_{HP} = -\frac{R_{f_2}}{R_{s_2}} = -10,$$

$$\omega_{LP} = \frac{1}{R_{f_1}C_{f_1}} = \frac{1}{10^5 \times 3.98 \times 10^{-11}} = 251.26 \text{ krad/s},$$

$$\omega_{HP} = \frac{1}{R_{s_2}C_{s_2}} = \frac{1}{10^4 \times 7.96 \times 10^{-11}} = 125.63 \text{ krad/s}.$$

Figure P9.38(a) displays the calculated plot of M [dB].

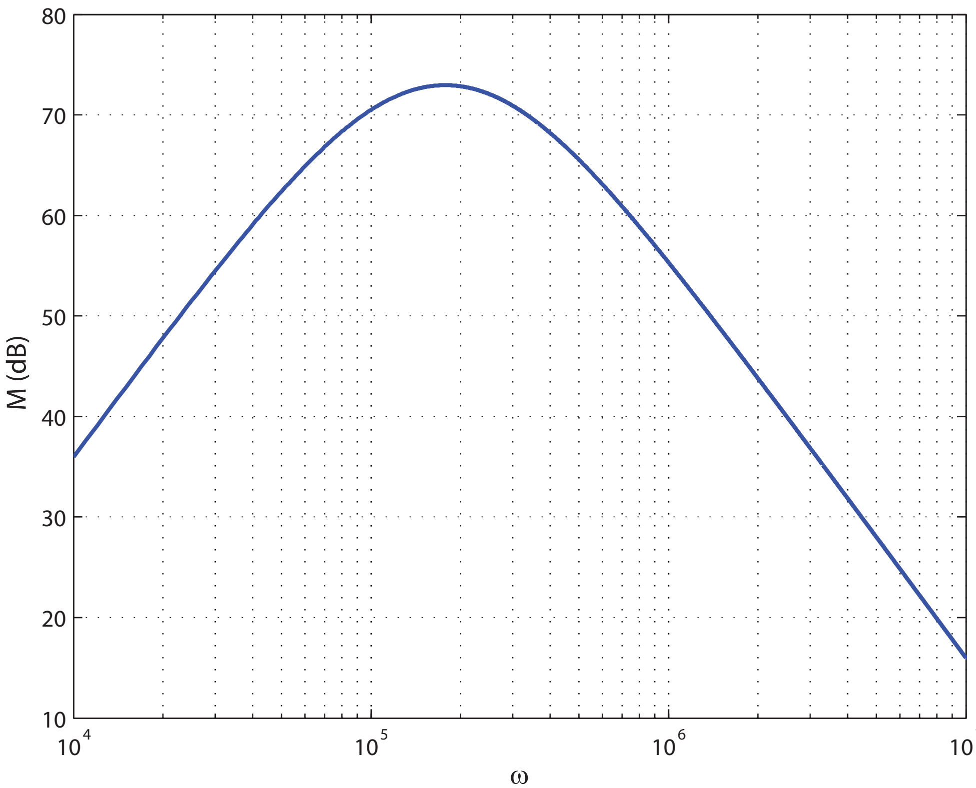


Figure P9.38(b)

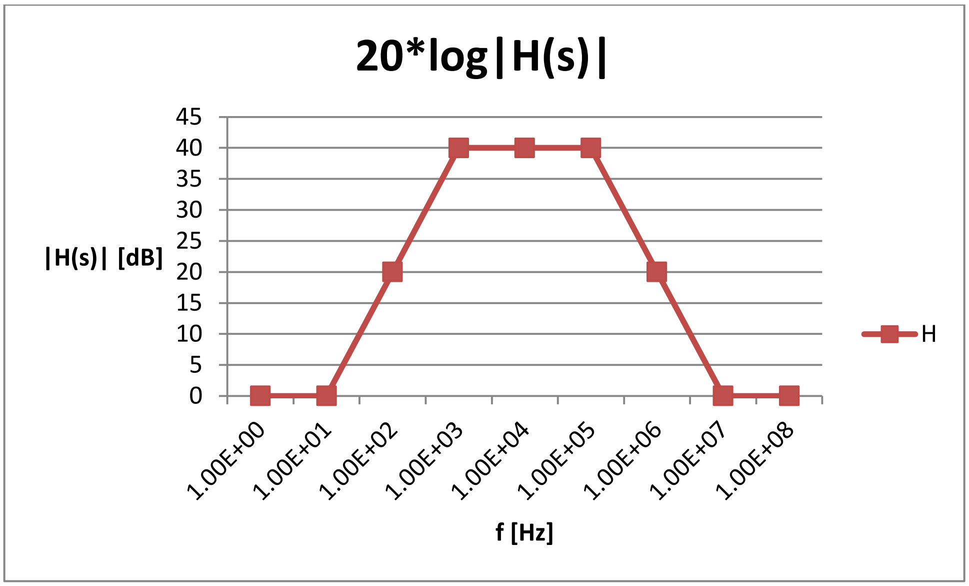
From the plot we determine that:

$$\omega_0 = 178 \text{ krad/s} \qquad (M(\omega_0) = 72.9563 \text{ dB}),$$
$$\omega_1(-3 \text{ dB}) = 94 \text{ krad/s},$$
$$\omega_2(-3 \text{ dB}) = 336 \text{ krad/s}.$$

2. (a) The given system $H(s)$, has:

- Zeros: $f_{z_1} = 10[Hz],\ f_{z_2} = 10^7[Hz]$
- Poles: $f_{p_1} = 10^3[Hz],\ f_{p_2} = 10^5[Hz]$

The slopes are $-\ 20\left[{dB}/{dec}\right]$



The actual values are calculated from the following formula –

$$|H(s)| = 20\cdot\left[\log\left(\sqrt{\frac{\omega^2}{4\pi^2\cdot 10^2}+1}\right)+\log\left(\sqrt{\frac{\omega^2}{4\pi^2\cdot 10^{14}}+1}\right)-\log\left(\sqrt{\frac{\omega^2}{4\pi^2\cdot 10^6}+1}\right)-\log\left(\sqrt{\frac{\omega^2}{4\pi^2\cdot 10^{10}}+1}\right)\right]=$$

$$20\cdot\left[\log\left(\sqrt{\frac{f^2}{10^2}+1}\right)+\log\left(\sqrt{\frac{f^2}{10^{14}}+1}\right)-\log\left(\sqrt{\frac{f^2}{10^6}+1}\right)-\log\left(\sqrt{\frac{f^2}{10^{10}}+1}\right)\right]$$

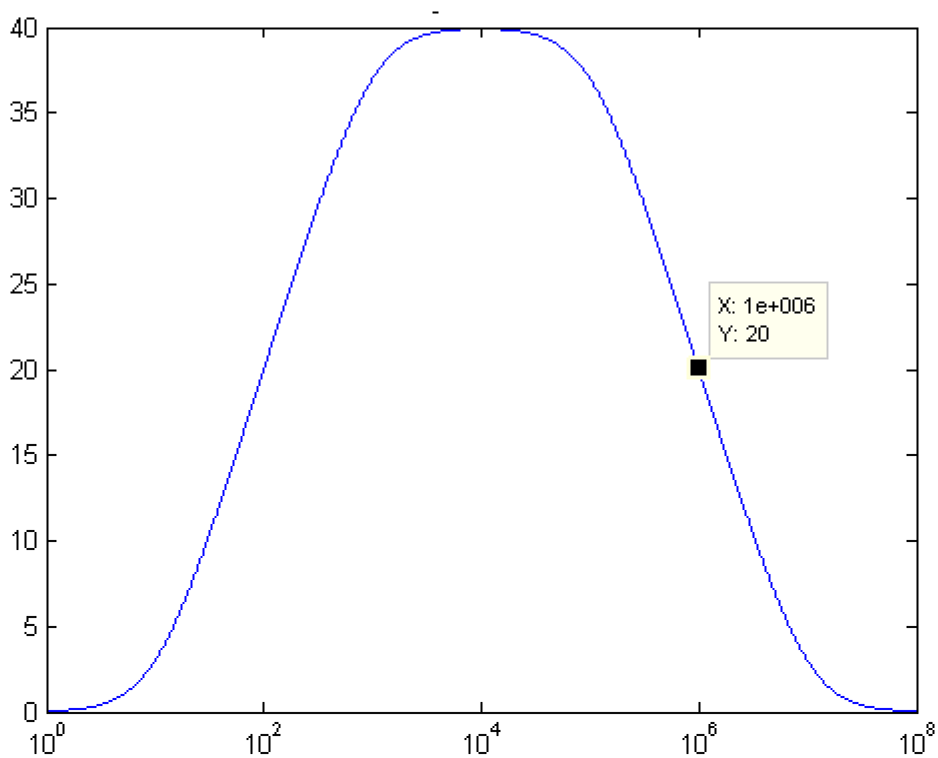
Values can be determined by plugging the frequency into the above formula, e.g. (marked on actual plot below):

$$|H(s=j2\pi\cdot 1MHz)| = 20\cdot\left[\log\left(\sqrt{\frac{10^{12}}{10^2}+1}\right)+\log\left(\sqrt{\frac{10^{12}}{10^{14}}+1}\right)-\log\left(\sqrt{\frac{10^{12}}{10^6}+1}\right)-\log\left(\sqrt{\frac{10^{12}}{10^{10}}+1}\right)\right]=$$

$$100[dB]+0.04[dB]-60[dB]-20.04[dB]=20[dB]$$

Magnitude [dB] vs. frequency [Hz])

Actual plot



(b) $\left|H\left(s = j2\pi \cdot 10Hz\right)\right| = 3.01[dB]$

$\left|H\left(s = j2\pi \cdot 10KHz\right)\right| = 39.91[dB]$

$\left|H\left(s = j2\pi \cdot 10MHz\right)\right| = 3.01[dB]$

3.

(a) Notice that the given input impedance is real, therefore it either consists of resistors only, or resistors in addition to a combination of capacitors and inductors which cancel each other at the center frequency.

$$R' = K_m \cdot R \Rightarrow K_m = \frac{20K\Omega}{1K\Omega} = 20$$

The center frequency is shifted, therefore –

$$\omega' = K_f \cdot \omega \Rightarrow K_f = \frac{100KHz}{5KHz} = 20$$

The net scaling factors for the components of the circuit are as following –

$$R' = K_m \cdot R = 20 \cdot R$$

$$C' = \frac{1}{K_m \cdot K_f} \cdot C = \frac{1}{400} \cdot C$$

$$L' = \frac{K_m}{K_f} \cdot L = L$$

In order for the quality factor to remain the same we require –

$$Q' = \frac{\omega_0'}{B'} = \frac{\omega_0}{B} = Q \Rightarrow \frac{2\pi \cdot 100KHz}{B'} = \frac{2\pi \cdot 5KHz}{500Hz}$$

$$\Rightarrow B' = 10KHz$$

$$\Rightarrow \omega_{c_1}' = 95KHz,\ \omega_{c_2}' = 105KHz$$

These values may be applied to different circuits, e.g. the example shown on page 441 in the book or others band-pass filters.

(b) Op-amp should be able to operate on high frequencies.
Op-amp should be as ideal as possible (High input resistance, low output resistance, high gain).

4. The general form of a transfer function with two zeros and two poles –

$$H(s) = \frac{K \cdot (s - s_{z_1})(s - s_{z_2})}{(s - s_{p_1})(s - s_{p_2})}$$

In the given problem we have the following –

- Zeros: $s_{z_1} = 0,\ s_{z_2} = -3$
- Poles: $s_{p_1} = -1 + 4j,\ s_{p_2} = -1 - 4j$

Therefore –

$$H(s) = \frac{K \cdot (s-0)(s+3)}{(s-(-1+4j))(s-(-1-4j))} = \frac{K \cdot s \cdot (s+3)}{(s-(-1+4j))(s-(-1-4j))}$$

Applying the given assumption –

$$\lim_{|s| \to \infty} H(s) = \frac{K \cdot s \cdot s}{s \cdot s} = 1 \Rightarrow K = 1$$

$$\Rightarrow H(s) = \frac{s \cdot (s+3)}{(s-(-1+4j))(s-(-1-4j))}$$