Section 8-2 and 8-3: Average and Complex Power

Problem 8.9 Determine the complex power, apparent power, average power absorbed, reactive power, and power factor (including whether it is leading or lagging) for a load circuit whose voltage and current at its input terminals are given by:

(a) $v(t) = 100\cos(377t - 30°)$ V,
$i(t) = 2.5\cos(377t - 60°)$ A.

(b) $v(t) = 25\cos(2\pi \times 10^3 t + 40°)$ V,
$i(t) = 0.2\cos(2\pi \times 10^3 t - 10°)$ A.

(c) $\mathbf{V}_{\text{rms}} = 110\angle 60°$ V, $\mathbf{I}_{\text{rms}} = 3\angle 45°$ A.

(d) $\mathbf{V}_{\text{rms}} = 440\angle 0°$ V, $\mathbf{I}_{\text{rms}} = 0.5\angle 75°$ A.

(e) $\mathbf{V}_{\text{rms}} = 12\angle 60°$ V, $\mathbf{I}_{\text{rms}} = 2\angle -30°$ A.

Solution:

(a)

$$\mathbf{V}_{\text{rms}} = \frac{100}{\sqrt{2}}\, e^{-j30°} \text{ V},$$

$$\mathbf{I}_{\text{rms}} = \frac{2.5}{\sqrt{2}}\, e^{-j60°} \text{ V}.$$

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = \frac{100}{\sqrt{2}}\, e^{-j30°} \times \frac{2.5}{\sqrt{2}}\, e^{j60°} = 125e^{j30°} \qquad \text{(VA)}$$

$$S = |\mathbf{S}| = 125 \text{ VA}$$

$$P_{\text{av}} = \mathfrak{Re}[\mathbf{S}] = 125\cos 30° = 108.25 \text{ W}$$

$$Q = \mathfrak{Im}[\mathbf{S}] = 125\sin 30° = 62.5 \text{ VAR}$$

$$\phi_s = \phi_v - \phi_i = -30° + 60° = 30° \quad \text{(hence } pf \text{ is lagging)}$$

$$pf = \cos 30° = 0.866.$$

(b)

$$\mathbf{V}_{\text{rms}} = \frac{25}{\sqrt{2}}\, e^{j40°} \text{ V},$$

$$\mathbf{I}_{\text{rms}} = \frac{0.2}{\sqrt{2}}\, e^{-j10°} \text{ A},$$

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = 2.5e^{j50°} \qquad \text{(VA)},$$

$$S = 2.5 \text{ VA}, \qquad \phi_s = 50° \text{ (lagging)}$$

$$P_{\text{av}} = 2.5\cos 50° = 1.61 \text{ W},$$

$$Q = 2.5\sin 50° = 1.92 \text{ VAR},$$

$$pf = \cos 50° = 0.64 \text{ lagging}.$$

(c)

$$\mathbf{S} = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}} = 110e^{j60°} \times 3e^{-j45°} = 330e^{j15°} \text{ VA}.$$

$$S = 330 \text{ VA},$$

$$P_{av} = 330 \cos 15^\circ = 318.76 \text{ W},$$
$$Q = 330 \sin 15^\circ = 85.41 \text{ VAR},$$
$$pf = \cos 15^\circ = 0.97 \text{ (lagging)}.$$

(d)

$$\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = 440 \times 0.5e^{-j75^\circ} = 220e^{-j75^\circ} \text{ VA}.$$
$$S = 220 \text{ VA},$$
$$P_{av} = 220 \cos(-75^\circ) = 56.94 \text{ W},$$
$$Q = 220 \sin(-75^\circ) = -212.50 \text{ VAR},$$
$$pf = \cos(-75^\circ) = 0.26 \text{ (leading)}.$$

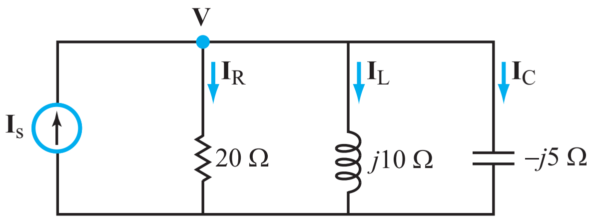
(e)

$$\mathbf{S} = \mathbf{V}_{rms}\mathbf{I}^*_{rms} = 12e^{j60^\circ} \times 2e^{j30^\circ} = 24e^{j90^\circ} \text{ VA}.$$
$$S = 24 \text{ VA},$$
$$P_{av} = 24 \cos 90^\circ = 0,$$
$$Q = 24 \sin 90^\circ = 24 \text{ VAR},$$
$$pf = \cos 90^\circ = 0 \quad \text{(purely inductive with } \mathbf{I} \text{ lagging } \mathbf{V} \text{ by } 90^\circ)$$

Problem 8.11 In the circuit of Fig. P8.11, $i_s(t) = 0.2 \sin 10^5 t$ A, $R = 20\ \Omega$, $L = 0.1$ mH, and $C = 2\ \mu$F. Show that the sum of the complex powers for the three passive elements is equal to the complex power of the source.

Figure P8.11: Circuit for Problem 8.11.

Solution:



$$\mathbf{I}_s = 0.2\angle 0^\circ \text{ A}$$

$$\mathbf{Z}_L = j\omega L = j10^5 \times 10^{-4} = j10\ \Omega$$

$$\mathbf{Z}_C = \frac{-j}{\omega C} = \frac{-j}{10^5 \times 2 \times 10^{-6}} = -j5\ \Omega.$$

$$\frac{\mathbf{V}}{20} + \frac{\mathbf{V}}{j10} + \frac{\mathbf{V}}{-j5} = \mathbf{I}_s = 0.2$$

$$\mathbf{V} = 1.79 e^{-j63.4\circ} \text{ V.}$$

$$\mathbf{I}_R = \frac{\mathbf{V}}{20} = \frac{1.79 e^{-j63.4^\circ}}{20} \text{ A.}$$

$$\mathbf{S}_R = \frac{1}{2}\mathbf{V}\mathbf{I}_R^* = \frac{(1.79)^2}{2 \times 20} = 0.08 \text{ VA.}$$

$$\mathbf{I}_L = \frac{\mathbf{V}}{j10} = \frac{1.79}{10}\ e^{-j153.4^\circ}$$

$$\mathbf{S}_L = \frac{1}{2}\mathbf{V}\mathbf{I}_L^* = \frac{1.79}{2}\ e^{-j63.4^\circ} \times \frac{1.79}{10}\ e^{j153.4^\circ} = 0.16 e^{j90^\circ} = 0 + j0.16 \text{ VA.}$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{-j5} = \frac{1.79}{5}\ e^{j26.6^\circ} \text{ A.}$$

$$\mathbf{S}_C = \frac{1}{2}\mathbf{V}\mathbf{I}_C^* = \frac{1}{2} 1.79 e^{-j63.4^\circ} \times \frac{1.79}{5}\ e^{-j26.6^\circ} = 0.32 e^{-j90^\circ} = 0 - j0.32 \text{ VA.}$$

$$\mathbf{S}_T = \mathbf{S}_R + \mathbf{S}_L + \mathbf{S}_C = 0.08 + j0.16 - j0.32 = 0.08 - j0.16 \text{ VA.}$$

For the source,

$$\mathbf{S}_s = \frac{1}{2}\mathbf{V}\mathbf{I}_s^* = \frac{1}{2}\ 1.79 e^{-j63.4^\circ} \times 0.2 = 0.179 e^{-j63.4^\circ}$$

$$= 0.179\cos 63.4^\circ - j0.179\sin 63.4^\circ$$
$$= 0.08 - j0.16 \text{ VA}.$$

Hence

$$\mathbf{S}_\mathrm{T} = \mathbf{S}_\mathrm{s}.$$

Problem 8.14 Determine **S** for the RL load in the circuit of Fig. P8.14, given that $\mathbf{I}_s = 4\angle 0^\circ$ A, $R_1 = 10\ \Omega$, $R_2 = 5\ \Omega$, $\mathbf{Z}_C = -j20\ \Omega$, $R = 10\ \Omega$, and $\mathbf{Z}_L = j20\ \Omega$.

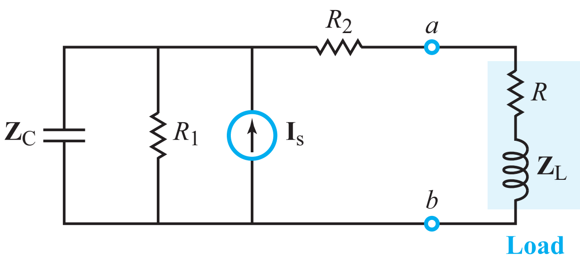
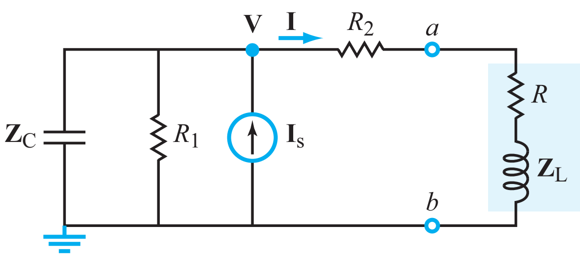


Figure P8.14: Circuit for Problem 8.14.

Solution:



$$\mathbf{V}\left(\frac{1}{\mathbf{Z}_C} + \frac{1}{R_1} + \frac{1}{R_2 + R + \mathbf{Z}_L}\right) = \mathbf{I}_s$$

$$\mathbf{V}\left(\frac{1}{-j20} + \frac{1}{10} + \frac{1}{5 + 10 + j20}\right) = 4$$

Solution gives

$$\mathbf{V} = 31.6 - j4.6 = 31.93e^{-j8.28^\circ}\ \text{V}.$$

For RL load:

$$\begin{aligned}
\mathbf{S} &= \frac{1}{2}\mathbf{V}_{ab}\mathbf{I}^* \\
&= \frac{1}{2}\,\frac{(10+j20)}{15+j20}\,\mathbf{V} \times \left(\frac{\mathbf{V}}{15+j20}\right)^* \\
&= (5+j10)\,\frac{|\mathbf{V}|^2}{|15+j20|^2} = 11.18e^{j63.4^\circ} \times \frac{(31.93)^2}{625} = 18.24e^{j63.4^\circ}\ \text{VA}.
\end{aligned}$$

Problem 8.20 The apparent power entering a certain load $\mathbf{Z}$ is 250 VA at a power factor of 0.8 leading. If the rms phasor voltage of the source is 125 V at 1 MHz:

(a) Determine $\mathbf{I}_{\text{rms}}$ going into the load

(b) Determine $\mathbf{S}$ into the load

(c) Determine $\mathbf{Z}$

(d) The equivalent impedance of the load circuit should be of the form $\mathbf{Z} = R + j\omega L$ or $\mathbf{Z} = R - j/\omega C$. Determine the value of L or C, whichever is applicable.

Solution:

(a) From

$$S = V_{\text{rms}} I_{\text{rms}},$$

$$I_{\text{rms}} = \frac{S}{V_{\text{rms}}} = \frac{250}{125} = 2 \text{ A}.$$

(b) $pf = 0.8$ leading means ϕ_z is negative. Hence,

$$\phi_z = -\cos^{-1} 0.8 = -36.87^\circ.$$

$$\begin{aligned}\mathbf{S} &= S\cos\phi_z + jS\sin\phi_z \\ &= 250[\cos(-36.87^\circ) + j\sin(-36.87^\circ)] = (200 - j150) \text{ VA}.\end{aligned}$$

(c)

$$P_{\text{av}} = I_{\text{rms}}^2 R$$

$$R = \frac{P_{\text{av}}}{I_{\text{rms}}^2} = \frac{200}{4} = 50\ \Omega.$$

Also,

$$Q = I_{\text{rms}}^2 X$$

$$X = \frac{Q}{I_{\text{rms}}^2} = \frac{-150}{4} = -37.5\ \Omega.$$

Hence,

$$Z = R + jX = (50 - j37.5)\ \Omega.$$

(d) $\dfrac{-j}{\omega C} = -j37.5$, or

$$C = \frac{1}{37.5\omega} = \frac{1}{37.5 \times 2\pi \times 10^6} = 4.24 \text{ nF}.$$

Section 8-4: Power Factor

Problem 8.22 The RL load in Fig. P8.22 is compensated by adding the shunt capacitance C so that the power factor of the combined (compensated) circuit is exactly unity. How is C related to R, L, and ω in that case?

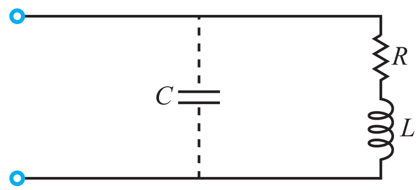


Figure P8.22: Circuit for Problem 8.22.

Solution: For the combined load, the impedance is

$$
\begin{aligned}
Z &= (R + j\omega L) \parallel \left(\frac{1}{j\omega C}\right) \\
&= \frac{(R + j\omega L)\left(\frac{-j}{\omega C}\right)}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \\
&= \frac{\omega L - jR}{\omega RC - j(1 - \omega^2 LC)} \\
&= \frac{(\omega L - jR)[\omega RC + j(1 - \omega^2 LC)]}{[\omega RC - j(1 - \omega^2 LC)][\omega RC + j(1 - \omega^2 LC)]} \\
&= \frac{[\omega^2 RLC + R(1 - \omega^2 LC)]}{\omega^2 R^2 C^2 + (1 - \omega^2 LC)^2} + \frac{j[\omega L(1 - \omega^2 LC) - \omega R^2 C]}{\omega^2 R^2 C^2 + (1 - \omega^2 LC)^2}
\end{aligned}
$$

For the pf to be unity, the imaginary component of **Z** has to be zero, which is realized if

$$\omega L(1 - \omega^2 LC) - \omega R^2 C = 0.$$

or

$$C = \frac{L}{R^2 + \omega^2 L^2} \, .$$

Problem 8.23 The generator circuit shown in Fig. P8.23 is connected to a distant load via a long coaxial transmission line. The overall circuit can be modeled as in Fig. P8.23(b), in which the transmission line is represented by an equivalent impedance $\mathbf{Z}_{\text{line}} = (5 + j2)\ \Omega$.

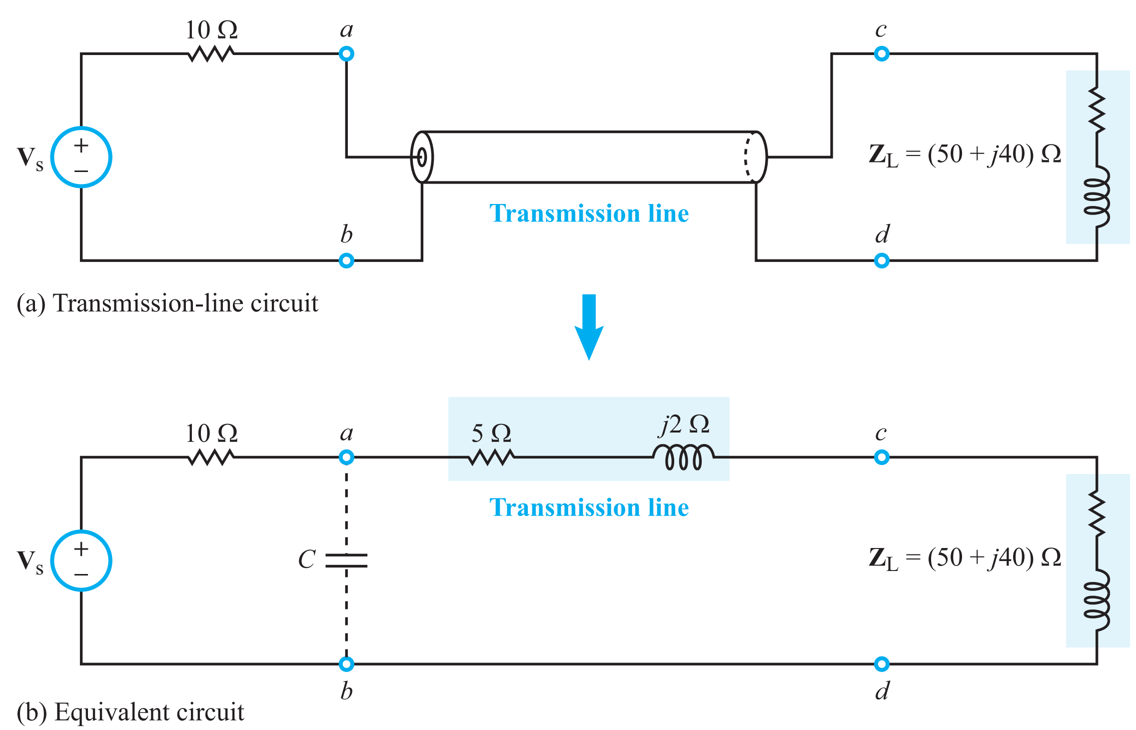


Figure P8.23: Circuit for Problem 8.23.

(a) Determine the power factor of voltage source $\mathbf{V}_s$.

(b) Specify the capacitance of a shunt capacitor C that would raise the power factor of the source to unity when connected between terminals (a,b). The source frequency is 1.5 kHz.

Solution:

(a) The power factor of the source is the same as the phase of the impedance representing the entire circuit connected to $\mathbf{V}_s$. Thus,

$$\mathbf{Z}_T = 10 + (5 + j2) + (50 + j40) = (65 + j42)\ \Omega,$$

$$\phi_Z = \tan^{-1}\frac{42}{65} = 32.87^\circ,$$

$$pf = \cos\phi_Z = 0.84,\ \text{lagging}.$$

(b) For the circuit to the right of terminals (a,b), the impedance—with a shunt capacitance C—is:

$$\mathbf{Z}_{ab} = \left(\frac{-j}{\omega C}\right) \parallel (55 + j42)$$

$$= \frac{-jX_C(55 + j42)}{55 + j(42 - X_C)}$$

where $X_C = \frac{1}{\omega C}$.

Simplifying,

$$\begin{aligned}\mathbf{Z}_{ab} &= \frac{42X_C - j55X_C}{55^2 + (42 - X_C)^2} \cdot [55 - j(42 - X_C)] \\ &= \frac{[42 \times 55X_C - 55X_C(42 - X_C)] - j[55^2 X_C + 42X_C(42 - X_C)]}{3025 + (42 - X_C)^2}\end{aligned}$$

For a *pf* of 1, the imaginary part of $\mathbf{Z}_{ab}$ should be zero,

$$55^2 X_C + 42X_C(42 - X_C) = 0.$$

Solution gives

$$X_C = 114.02,$$

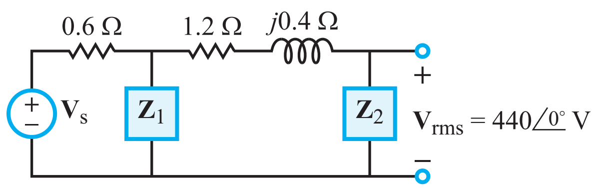
or

$$C = \frac{1}{\omega X_C} = \frac{1}{2\pi \times 1.5 \times 10^3 \times 114.02} = 930.5 \text{ nF}.$$

Problem 8.25 Use the power information given for the circuit in Fig. P8.25 to determine:

(a) $\mathbf{Z}_1$ and $\mathbf{Z}_2$

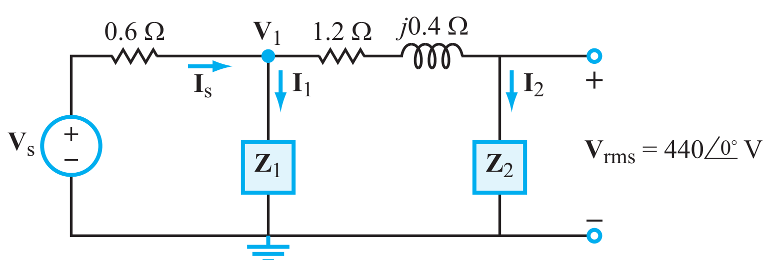
(b) the rms value of $\mathbf{V}_s$.



Load $\mathbf{Z}_1$: 24 kW @ pf = 0.66 leading
Load $\mathbf{Z}_2$: 18 kW @ pf = 0.82 lagging

Figure P8.25: Circuit for Problem 8.25.

Solution:



(a) For load $\mathbf{Z}_2$:

$$\phi_{Z_2} = \cos^{-1} 0.82 = 34.9°.$$

Since $\phi_{Z_2} = \phi_{v_2} - \phi_{i_2}$, and $\phi_{v_2} = 0$, $\quad\Longrightarrow\quad \phi_{i_2} = -34.9°.$

$$P_{\text{av}_2} = V_{\text{rms}} I_{2_{\text{rms}}} \cos \phi_{Z_2}$$

$$18 \times 10^3 = 440 I_{2_{\text{rms}}} \cos 34.9°,$$

or

$$I_{2_{\text{rms}}} = 49.88 \text{ A},$$

and

$$S_2 = V_{\text{rms}} I_{2_{\text{rms}}} = 440 \times 49.88 = 21.95 \text{ kVA}.$$

Also, $\mathbf{I}_{2_{\text{rms}}} = 49.88\angle{-34.9°}$ A.

If $\mathbf{Z}_2 = R_2 + jX_2$,

$$P_{\text{av}_2} = I_{2_{\text{rms}}}^2 R_2 \quad\Longrightarrow\quad R_2 = \frac{18 \times 10^3}{(49.88)^2} = 7.23\ \Omega$$

$$Q_2 = S_2 \sin \phi_{Z_2} = 21.95 \times 10^3 \sin 34.9° = 12.56 \text{ kVAR}$$

But

$$Q_2 = I_{2_{\text{rms}}}^2 X_2 \quad\Longrightarrow\quad X_2 = \frac{12.56 \times 10^3}{(49.88)^2} = 5.05\ \Omega.$$

Hence,

$$\mathbf{Z}_2 = (7.23 + j5.05)\ \Omega.$$

To determine $\mathbf{Z}_1$, we first determine the voltage across it, $\mathbf{V}_{1_{\rm rms}}$:

$$\begin{aligned}
\mathbf{V}_{1_{\rm rms}} &= (1.2 + j0.4 + \mathbf{Z}_2)\mathbf{I}_{2_{\rm rms}} \\
&= (1.2 + j0.4 + 7.23 + j5.05)49.88e^{-j34.9^\circ} \\
&= 500.8\angle{-2^\circ}\ \text{V}.
\end{aligned}$$

For load $\mathbf{Z}_1$:

$$\begin{aligned}
\phi_{Z_1} &= -\cos^{-1} 0.66 = -48.7^\circ \\
S_1 &= \frac{P_{\rm av_1}}{\cos\phi_{Z_1}} = \frac{24}{0.66} = 36.36\ \text{kVA} \\
I_{1_{\rm rms}} &= \frac{S_1}{V_{1_{\rm rms}}} = \frac{36.36 \times 10^3}{500.8} = 72.6\ \text{A} \\
\phi_{Z_1} &= \phi_{v_1} - \phi_{i_1} \\
-48.7^\circ &= -2 - \phi_{i_1} \quad \Longrightarrow \quad \phi_{i_1} = 46.7^\circ. \\
\mathbf{I}_{1_{\rm rms}} &= 72.6\angle{46.7^\circ}\ \text{A}. \\
P_{\rm av_1} &= \mathbf{I}^2_{1_{\rm rms}} R_1 \quad \Longrightarrow \quad R_1 = \frac{24 \times 10^3}{(72.6)^2} = 4.55\ \Omega \\
Q_1 &= S_1 \sin\phi_{Z_1} = 36.36 \times 10^3 \sin(-48.7^\circ) = -27.32\ \text{kVAR} \\
X_1 &= \frac{Q_1}{I^2_{1_{\rm rms}}} = \frac{-27.32 \times 10^3}{(72.6)^2} = -5.18\ \Omega
\end{aligned}$$

Hence,

$$\mathbf{Z}_1 = (4.55 - j5.18)\ \Omega.$$

(b) Given $\mathbf{I}_1$ and $\mathbf{I}_2$, we can now determine $\mathbf{I}_s$:

$$\begin{aligned}
\mathbf{I}_{s_{\rm rms}} &= \mathbf{I}_{1_{\rm rms}} + \mathbf{I}_{2_{\rm rms}} \\
&= 72.6e^{j46.7^\circ} + 49.88e^{-j34.9^\circ} \\
&= (90.7 + j24.3)\ \text{A} \\
\mathbf{V}_{s_{\rm rms}} &= \mathbf{V}_{1_{\rm rms}} + 0.6\mathbf{I}_{s_{\rm rms}} \\
&= 500.8e^{-j2^\circ} + 0.6(90.7 + j24.3) = (554.9 - j2.9)\ \text{V}.
\end{aligned}$$

Problem 8.27 For the circuit in Fig. P8.27, choose the load impedance $\mathbf{Z}_L$ so that the power dissipated in it is a maximum. How much power will that be?

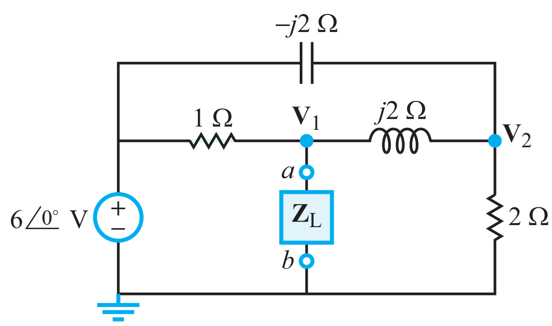
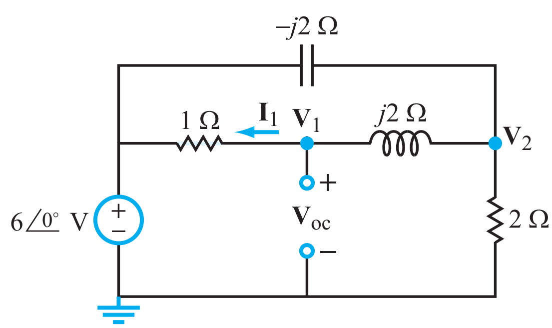


Figure P8.27: Circuit for Problem 8.27.

Solution: To determine $\mathbf{V}_s$ of the equivalent source circuit, we remove $\mathbf{Z}_L$ and calculate $\mathbf{V}_{oc}$ at terminals (a, b).



At node $\mathbf{V}_2$:

$$\frac{\mathbf{V}_2 - 6}{-j2} + \frac{\mathbf{V}_2}{2} + \frac{\mathbf{V}_2 - 6}{1 + j2} = 0 \qquad \Longrightarrow \qquad \mathbf{V}_2 = 0.6(3 + j) \text{ V}.$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_2 - 6}{1 + j2} = \frac{\mathbf{V}_1 - 6}{1}$$

Hence,

$$\mathbf{V}_s = \mathbf{V}_1 = \frac{\mathbf{V}_2 - 6}{1 + j2} + 6 = \frac{0.6(3 + j) - 6}{1 + j2} + 6 = 5.7\angle 18.4^\circ \text{ V}.$$

To determine $\mathbf{Z}_{Th}$ at terminals (a, b), we suppress the 6-V source and simplify the circuit. The process leads to:

$$\mathbf{Z}_s = \mathbf{Z}_{Th} = (0.6 + j0.2)\ \Omega$$

For maximum power transfer:

$$\mathbf{Z}_L = \mathbf{Z}_s^* = (0.6 - j0.2)\ \Omega,$$

and

$$P_{av}(\max) = \frac{1}{8}\frac{|\mathbf{V}_s|^2}{R_L} = \frac{1}{8} \times \frac{(4.9)^2}{0.6} = 6.78 \text{ W}.$$